PENGOLAHAN SINYAL DIGITAL

Modul 2.

Proses ADC-DAC

PROGRAM SARJANA FAKULTAS TEKNIK
(Undergraduate Program of Engineering School)

Telkom University

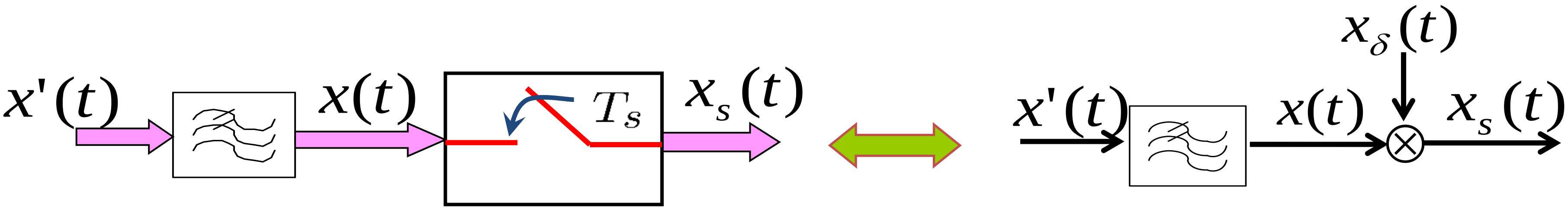
Content

- Konsep Sampling
- Kuantisasi
- Coding
- Decoding
- Filtering (ADC-DAC)
- Perhitungan error kuantisasi dikaitkan dengan level kuantisasi dan sampling rate

ADC (Analog to Digital Converter)

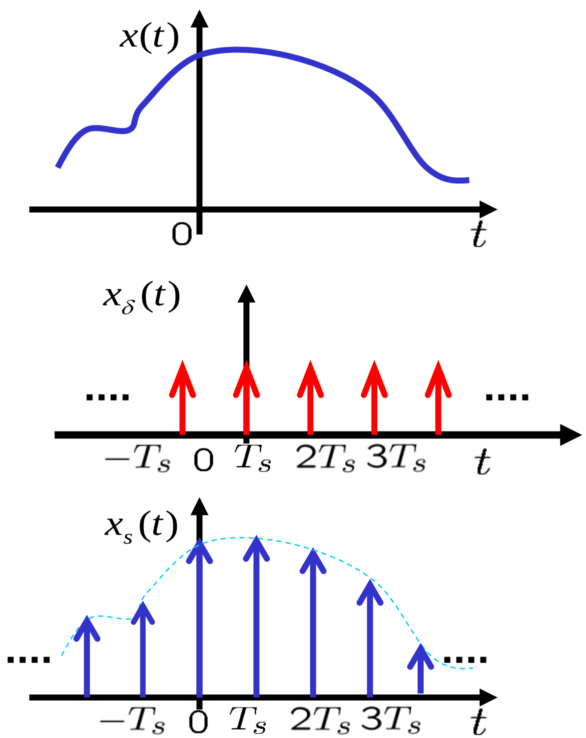
- Mengubah sinyal analog menjadi sinyal digital
- Proses yang terjadi dalam ADC :
  - Sampling (pencuplikan)
  - Quantizing (kuantiasasi)
  - Encoding (pengkodean)

Proses Pencuplikan (Sampling)



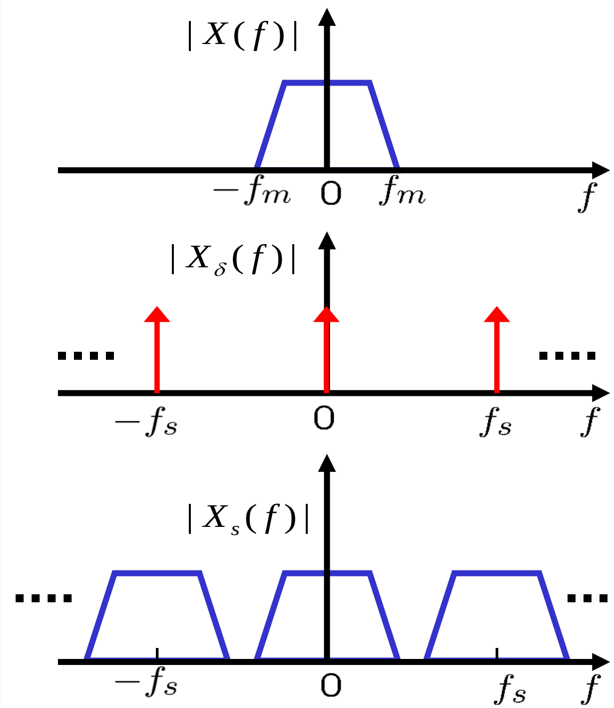
Time domain

$$x_s(t) = x_\delta(t) \times x(t)$$

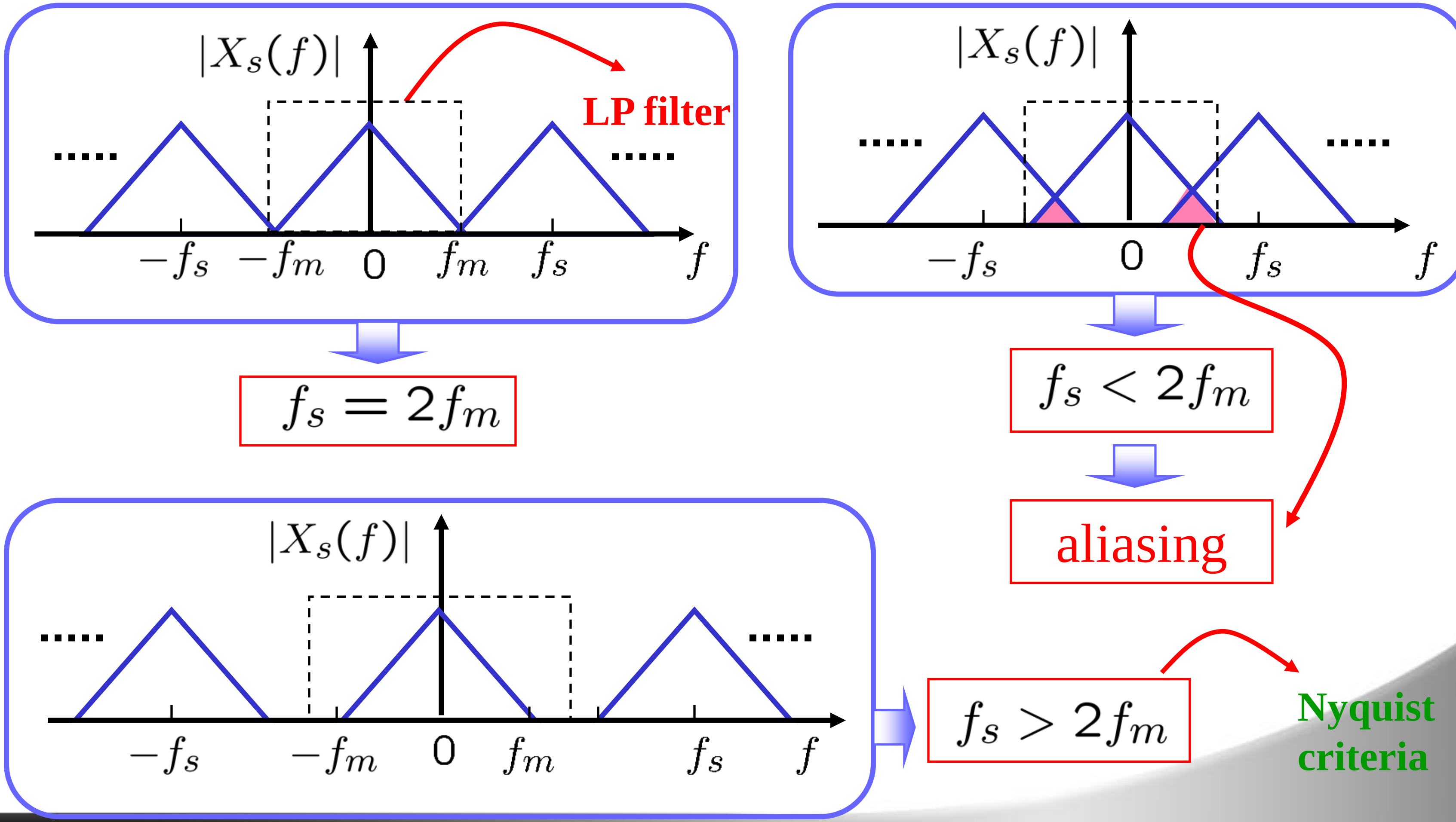


Frequency domain

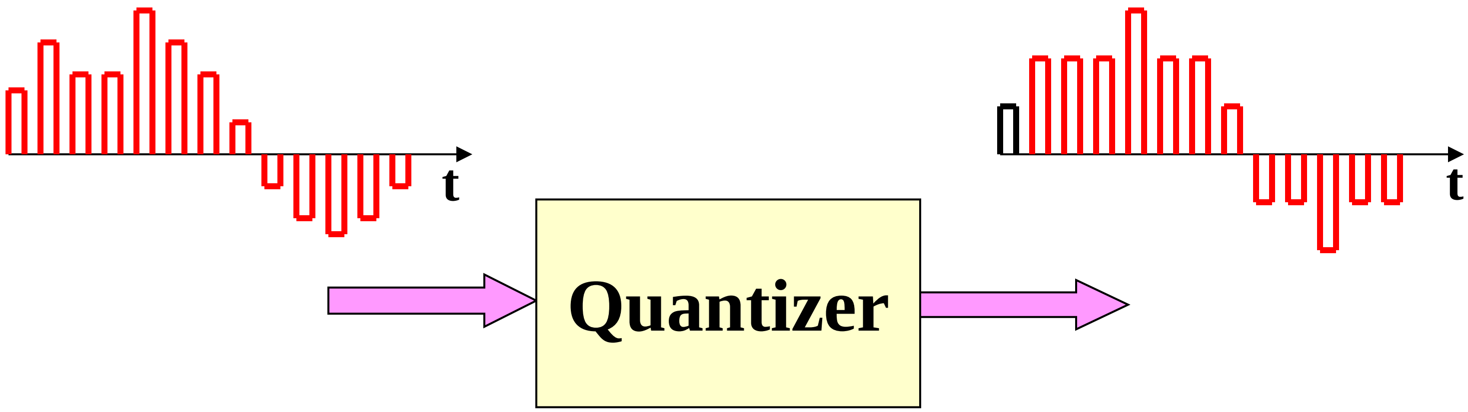
$$X_s(f) = X_\delta(f) * X(f)$$



ALIASING EFFECT



PROSES KUANTISASI (QUANTIZATION)



Kuantisasi : mengubah level amplituda menjadi diskret dengan jumlah terbatas.

Jumlah level kuantisasi $M = 2^N$, N = jumlah bit pengkodean

Terdapat 2 jenis kuantiser yaitu :

1) Kuantiser Uniform (lebar selang kuantisasi seragam)
2) Kuantiser Non-Uniform (lebar selang kuantisasi tidak seragam)

QUANTISER UNIFORM

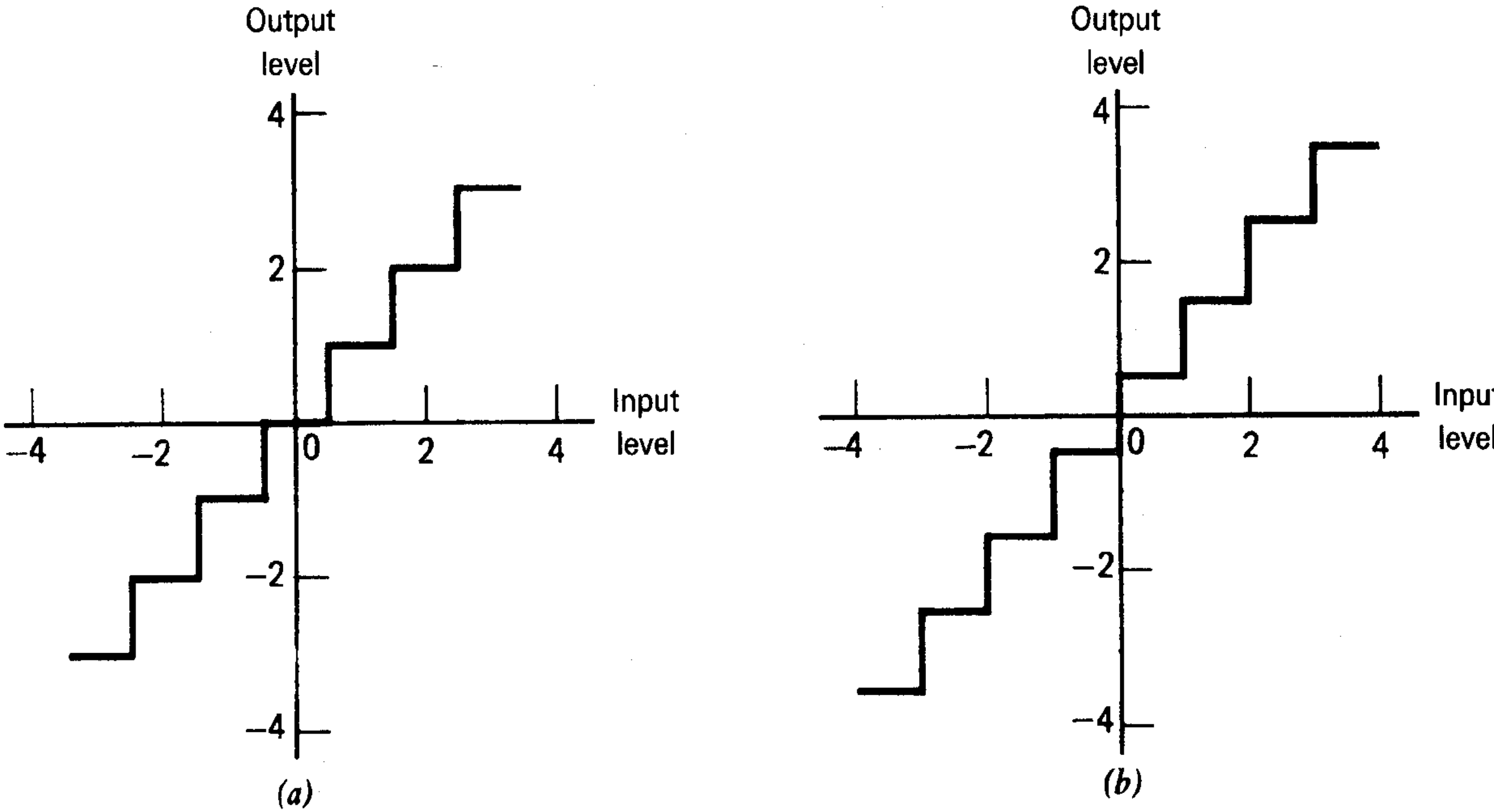
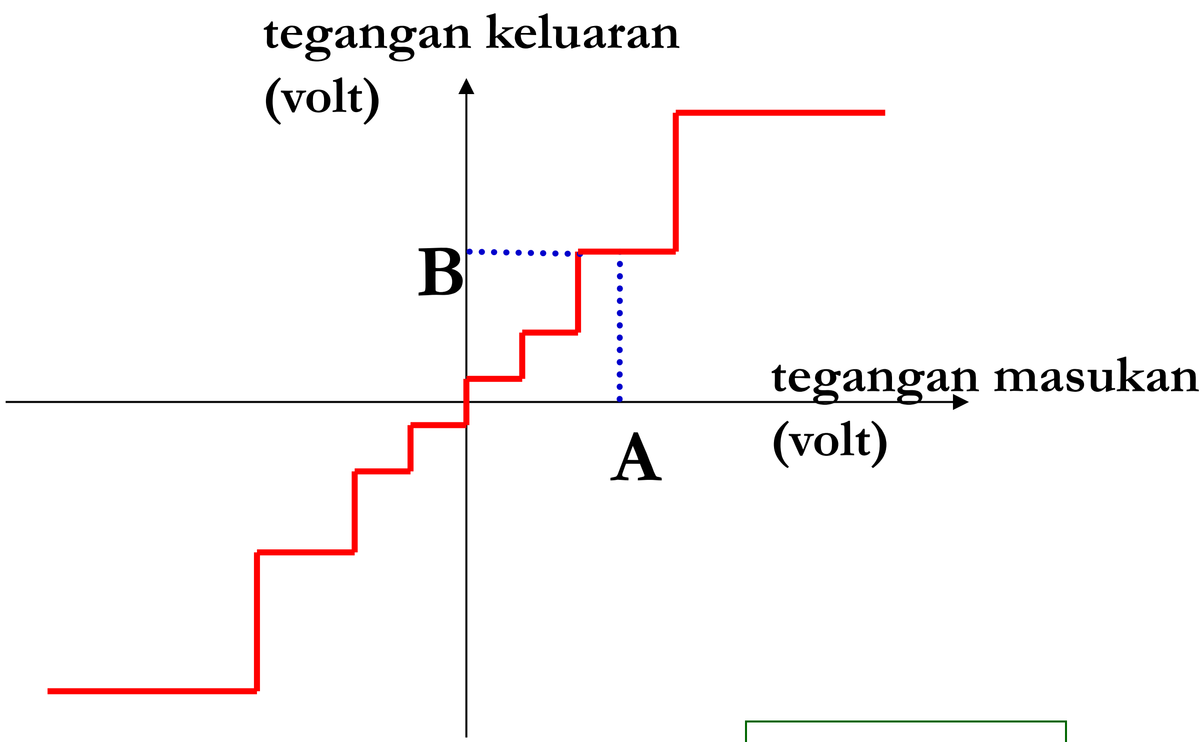
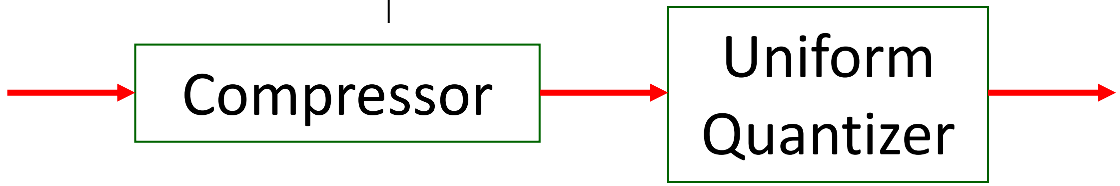


Figure 6.17 Two types of quantization: (*a*) midtread and (*b*) midrise.

QUANTISER NON-UNIFORM



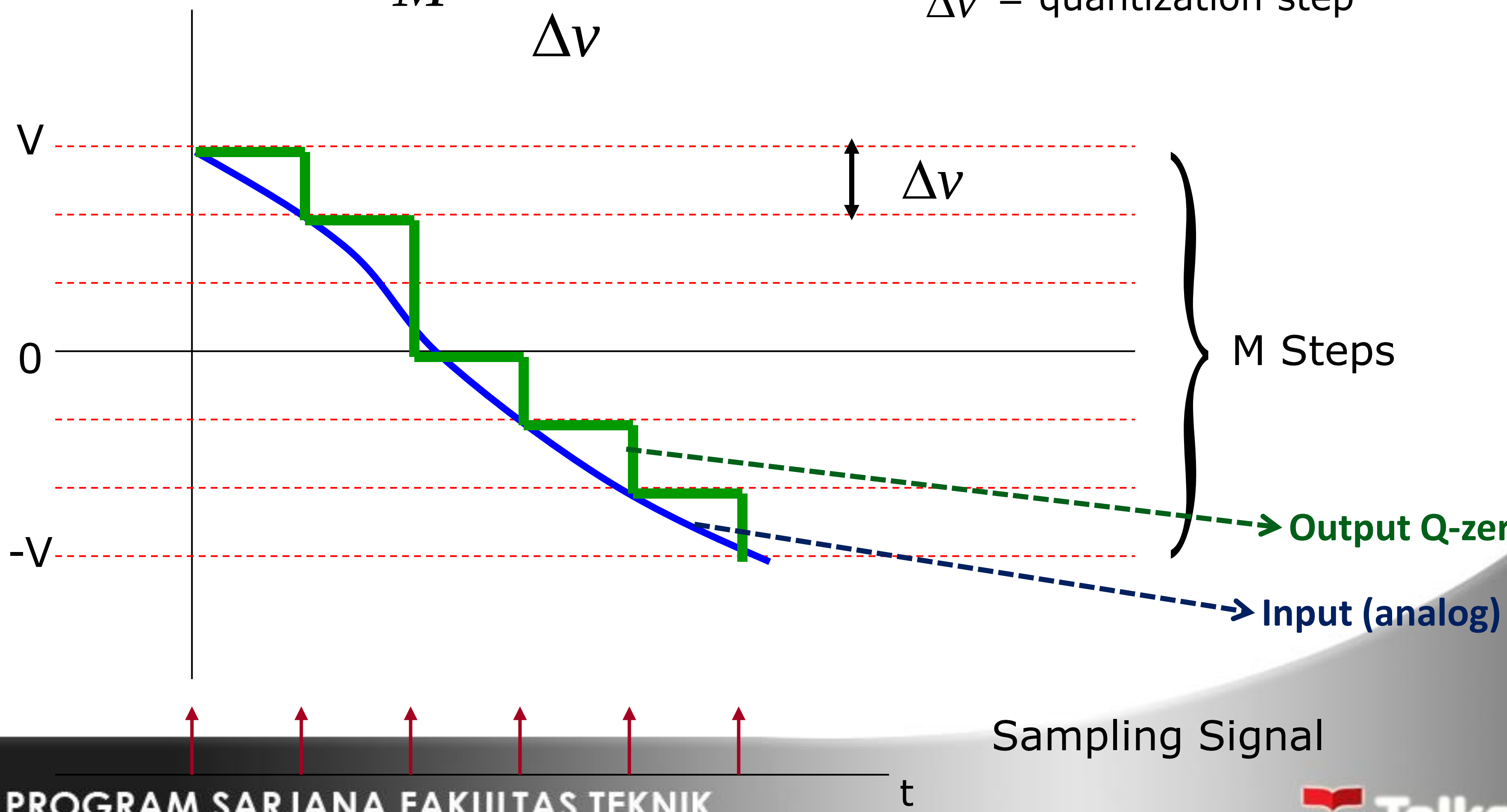


NonUniform / Nonlinear Quantizer

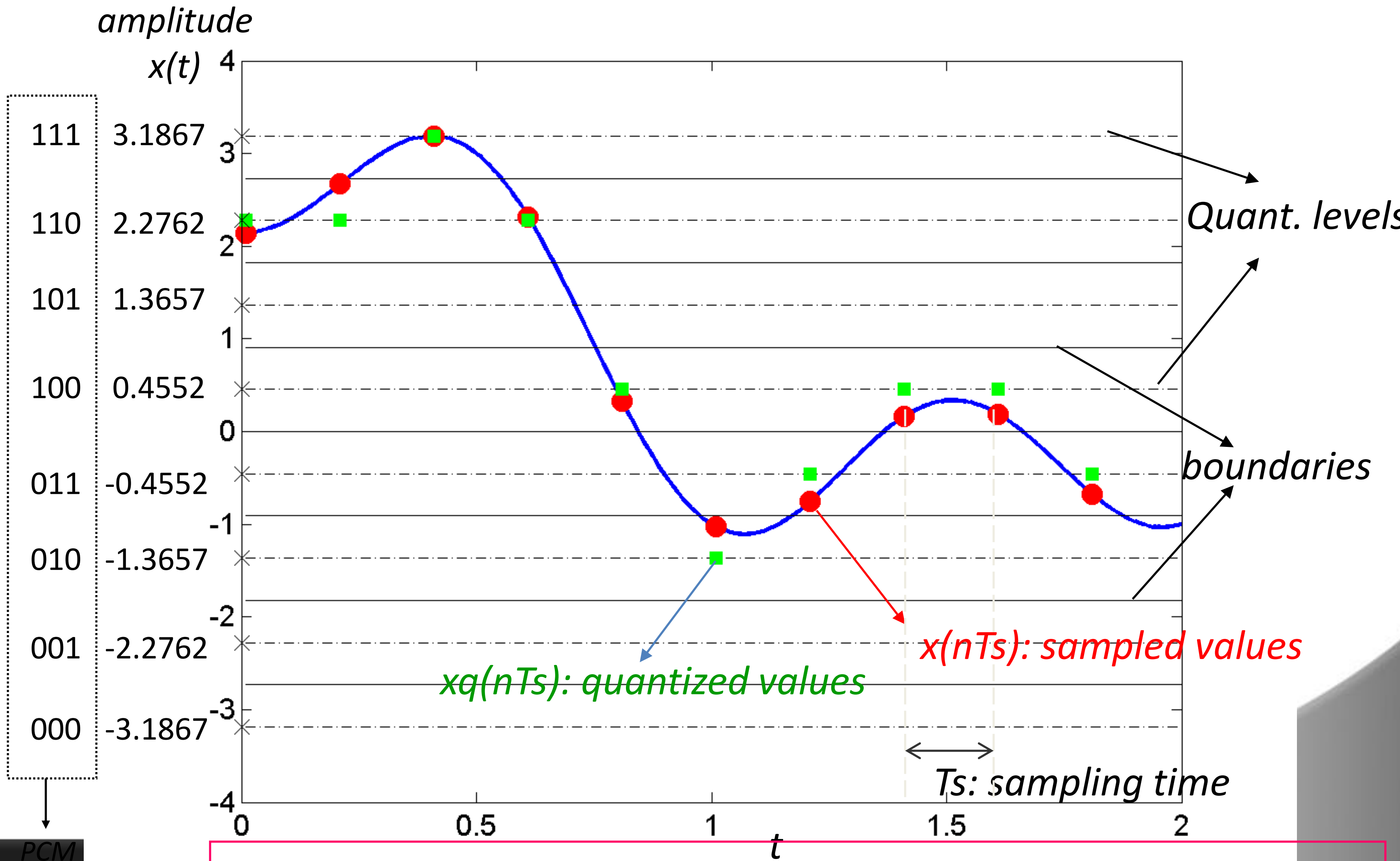
QUANTIZATION

$$M = \frac{2V}{\Delta v}$$

Where M = no. of steps
Δv = quantization step

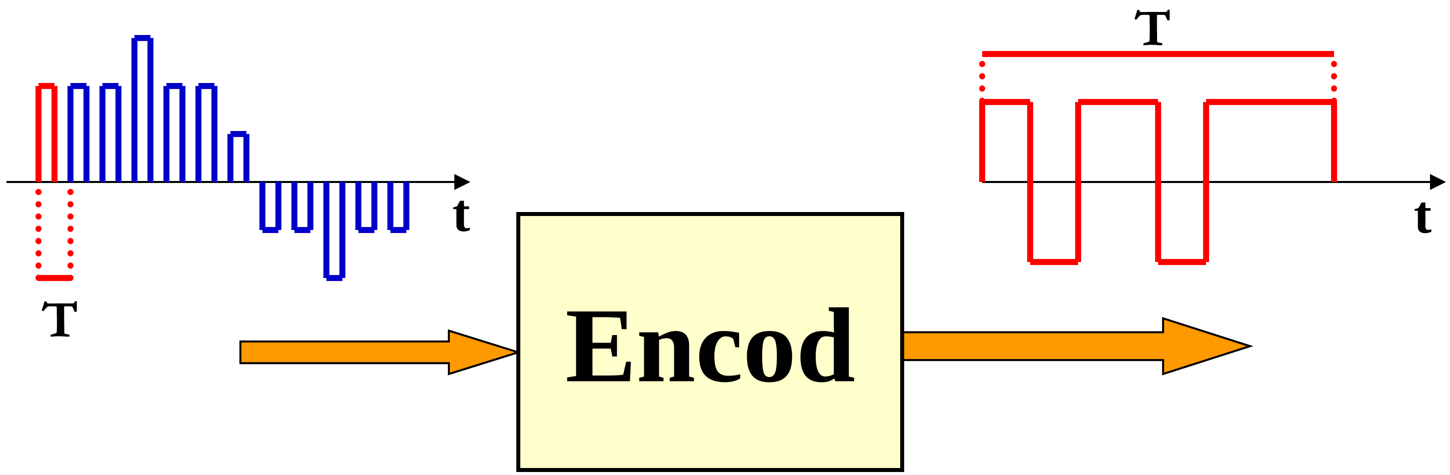


Quantization example



110 110 111 110 100 010 011 100 100 011 PCM sequence

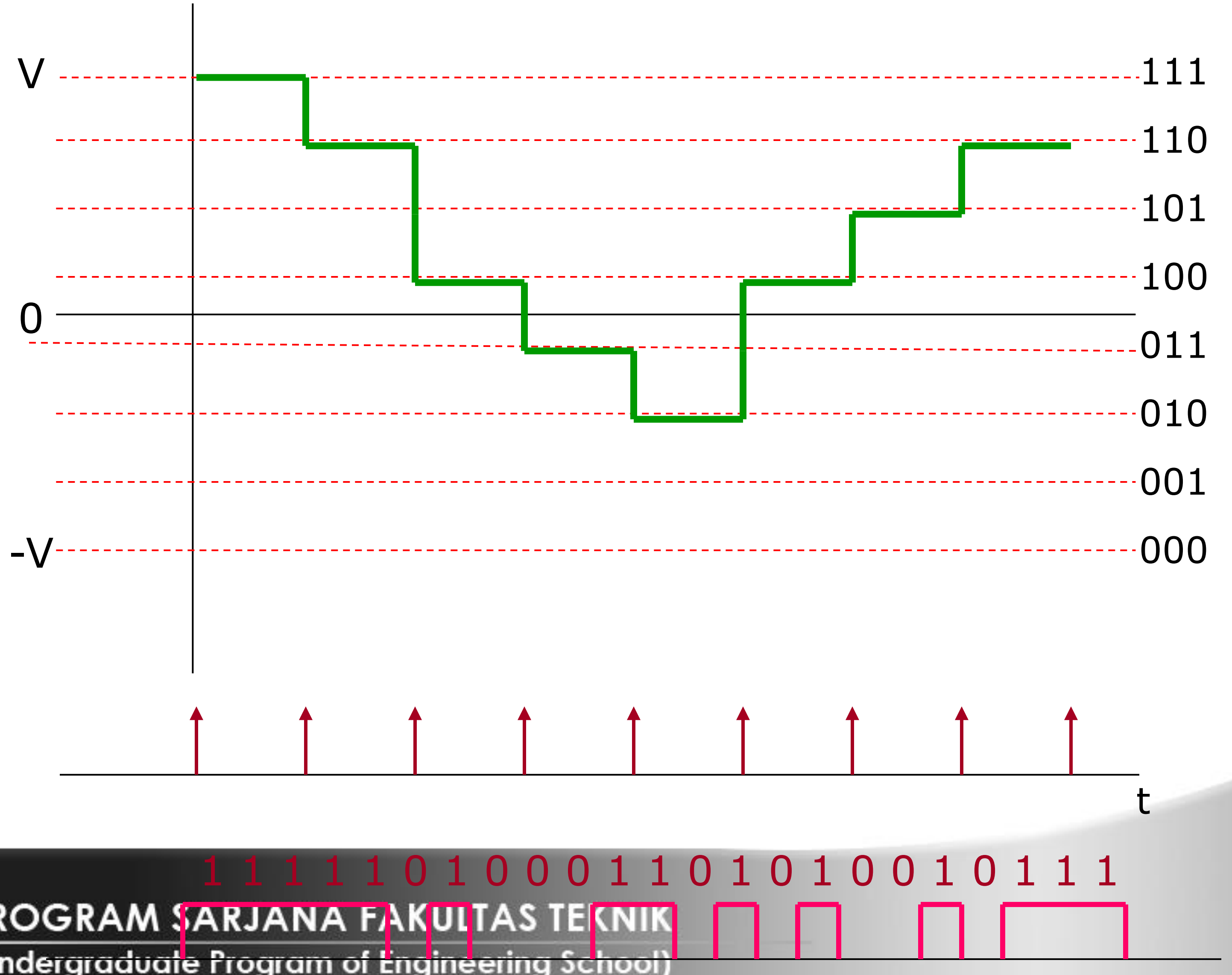
PROSES PENGKODEAN (ENCODING)



Contoh di atas menunjukkan proses encoding,
1 simbol masukan dikodekan menjadi 8 bit

Jumlah bit untuk mengkodekan tiap simbol ditentukan oleh perangkat ADC (Analog to Digital Converter)

ENCODING



$$M = 2^N$$

ERROR KUANTISASI

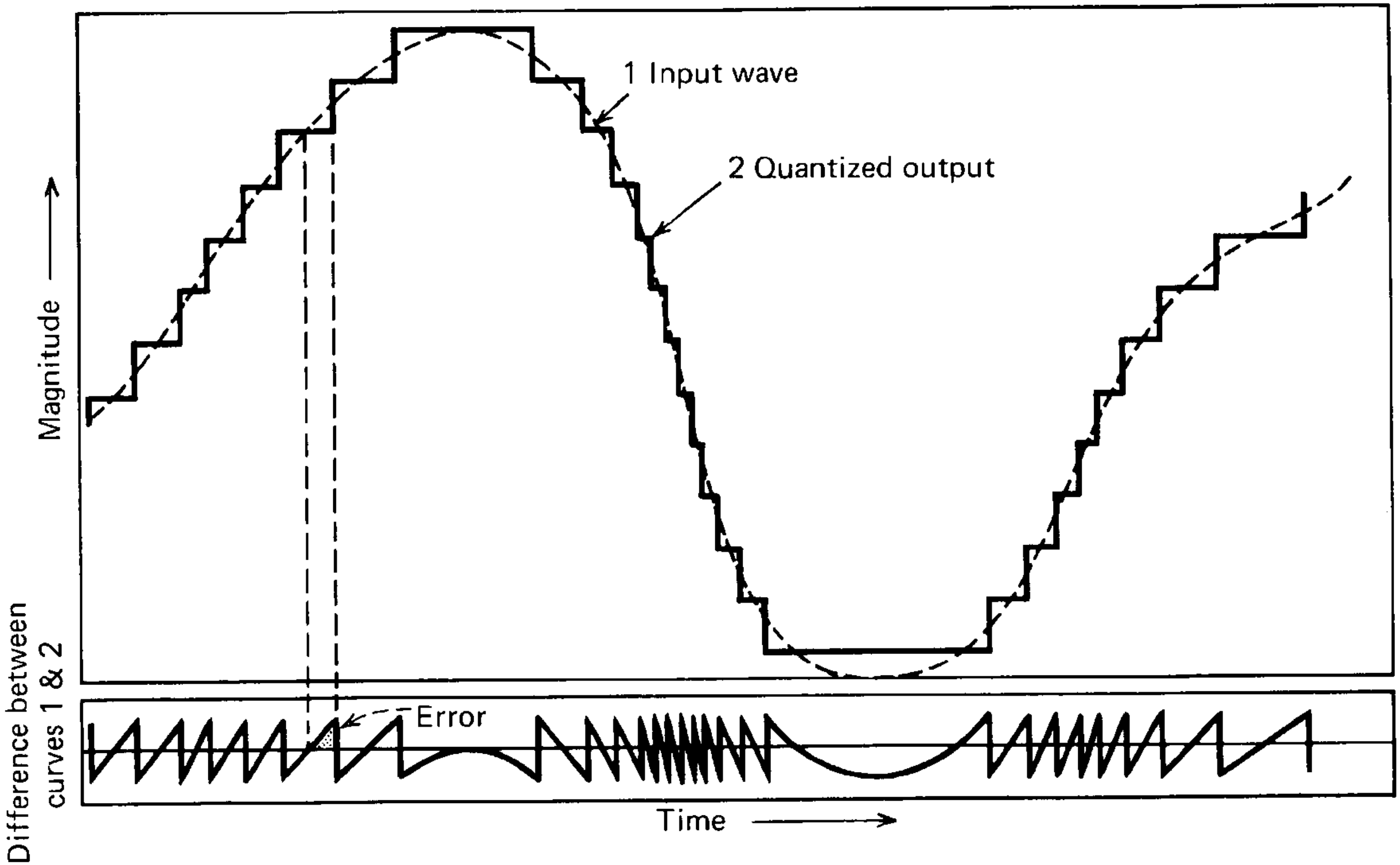
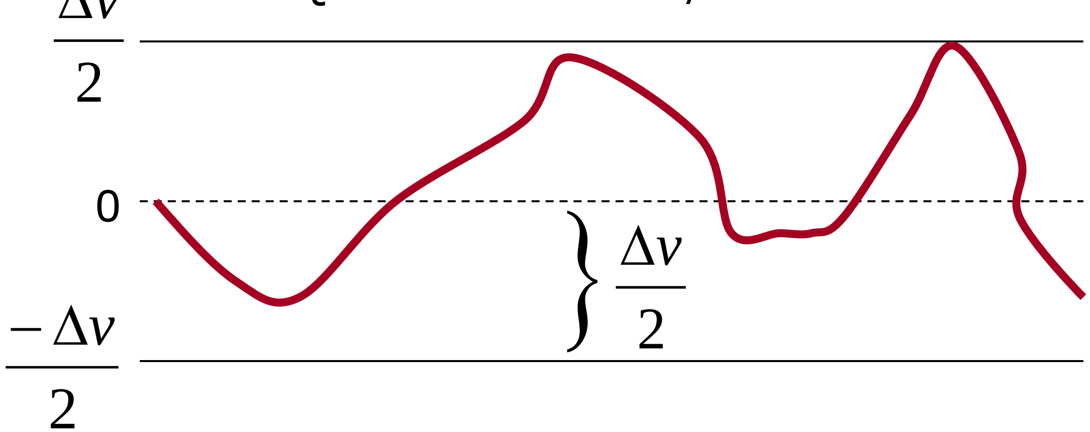


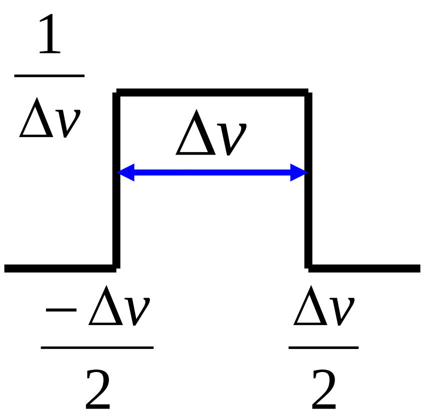
Figure 6.18 Illustration of the quantization process. (Adapted from Bennett, 1948, with permission of AT&T.)

ERROR KUANTISASI

Quantization Error/Noise



Uniform distribution



$$e(t) = f(t) - f_Q(t)$$

$$\frac{-\Delta v}{2} \leq e(t) \leq \frac{\Delta v}{2}$$

KONSEP FREKUENSI

Sinyal sinusoidal waktu kontinu

$$x_a(t) = A\cos(\Omega t + \theta) \qquad -\infty < t < \infty$$

t = waktu

A = amplituda

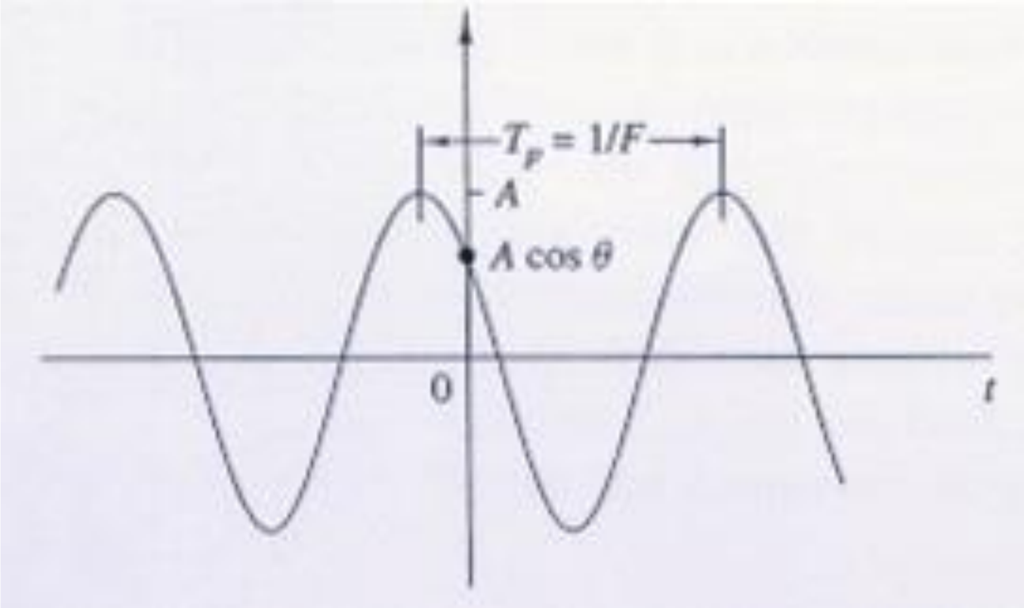
Ω = frekuensi sudut[radian/detik]

θ = fasa [radian]

$$\Omega = 2\pi F \quad \rightarrow \quad x_a(t) = A\cos(2\pi F t + \theta)$$

F = frekuensi [siklus/detik, hertz (Hz)]

$$x_a(t) = A\cos(\Omega t + \theta)$$



- **Untuk setiap frekuensi F → $x_a(t)$ periodik**

$$x_a(t + T_p) = x_a(t) \qquad T_p = \frac{1}{F} = \text{perioda dasar}$$

- **Sinyal-sinyal sinusoidal waktu kontinu dengan frekuensi berbeda dapat dibedakan**
- **Frekuensi diperbesar**

Untuk suatu waktu tertentu jumlah perioda bertambah

Sinyal sinusoidal waktu diskrit

$$x(n) = A\cos(\omega n + \theta) \qquad -\infty < n < \infty$$

n = bilangan bulat (integer)

A = amplituda

ω = frekuensi [radian/sampel]

θ = fasa [radian]

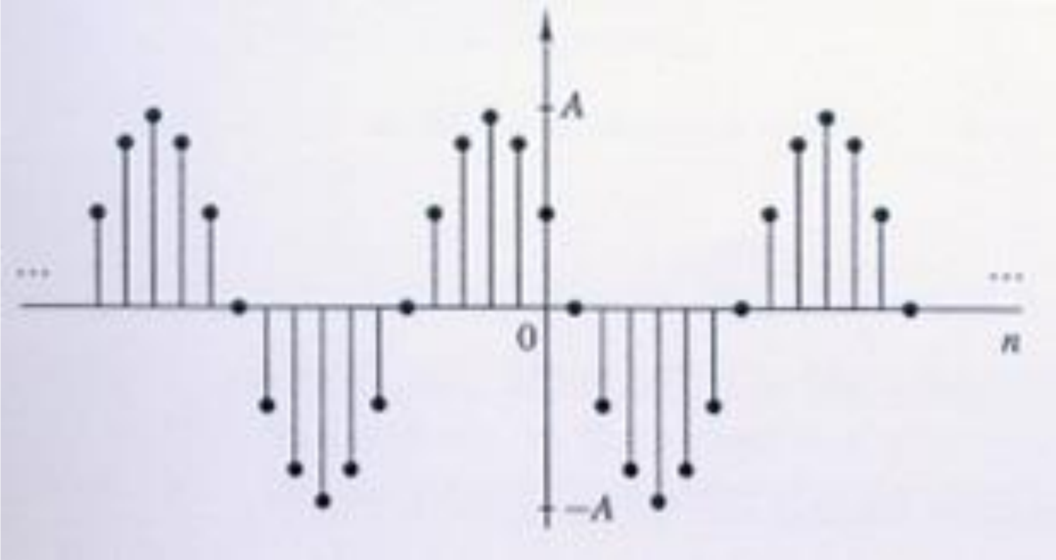
$$\omega = 2\pi f \quad \rightarrow \quad x(n) = A\cos(2\pi f\, n + \theta)$$

f = frekuensi [siklus/sampel]

$$x(n) = A\cos(2\pi f_o n + \theta)$$

$$\omega_o = \frac{\pi}{6} \quad \rightarrow \quad f_o = \frac{1}{12}$$

$$\theta = \frac{\pi}{3}$$



- x (n) periodik hanya bila frekuensi f merupakan bilangan rasional

$$x(n + N) = x(n)$$

$$\cos[2\pi f_o(n + N) + \theta] = \cos[2\pi f_o n + 2\pi f_o N + \theta] = \cos(2\pi f_o n + \theta)$$

$$2\pi f_o N = 2\pi k \quad \rightarrow \quad f_o = \frac{k}{N}$$

Harga terkecil dari N disebut perioda dasar

- **Sinyal-sinyal sinusoidal waktu diskrit dengan frekuensi-frekuensi yang berbeda sebanyak 2π k adalah identik (tidak dapat dibedakan)**

$$\cos[(\omega_o + 2\pi)n + \theta] = \cos[\omega_o n + 2\pi n + \theta] = \cos(\omega_o n + \theta)$$

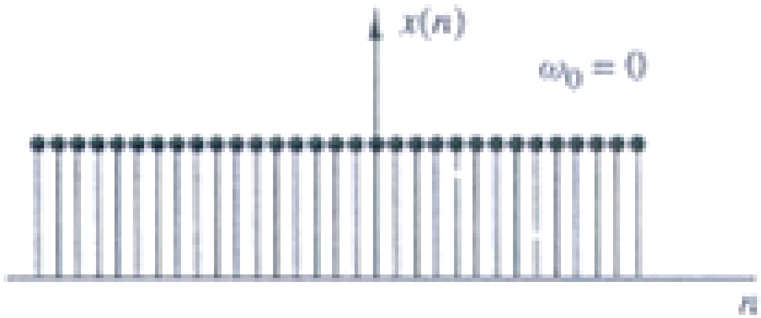
$$x_k(n) = A\cos(\omega_k n + \theta) \quad k = 0, 1, 2 \cdots$$

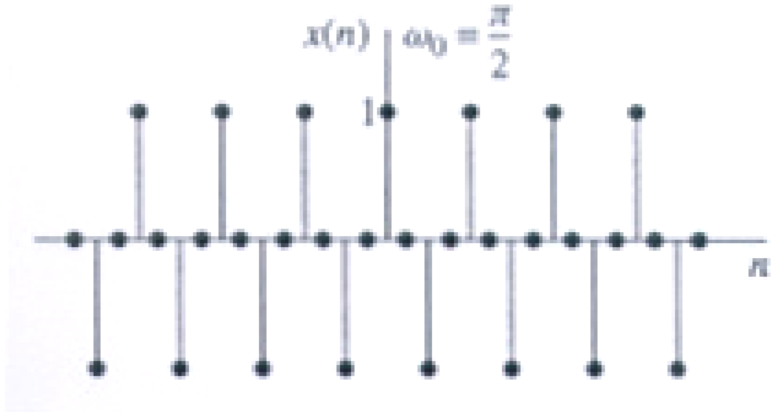
$$\omega_k = \omega_o + 2\pi k$$

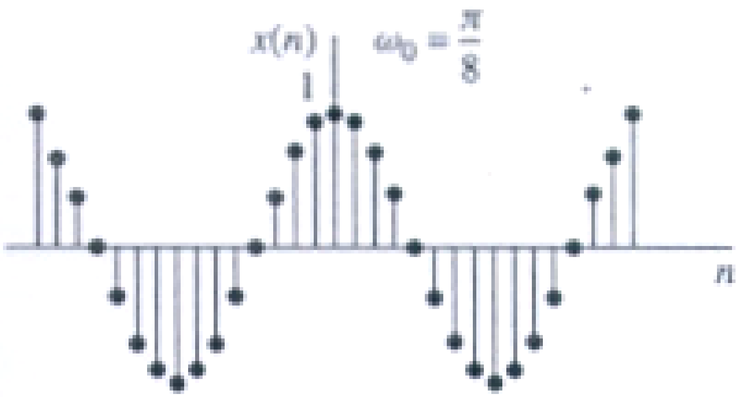
$$-\pi < \omega < \pi \quad \rightarrow \quad -\frac{1}{2} < f < \frac{1}{2}$$

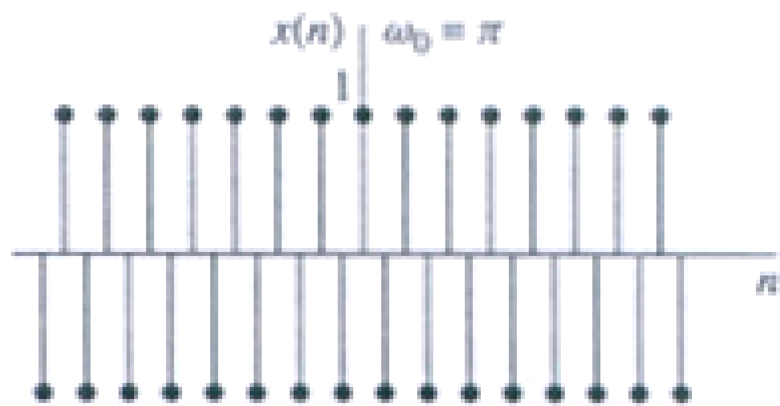
- **Frekuensi diperbesar → harga maksimum f = 1/2**

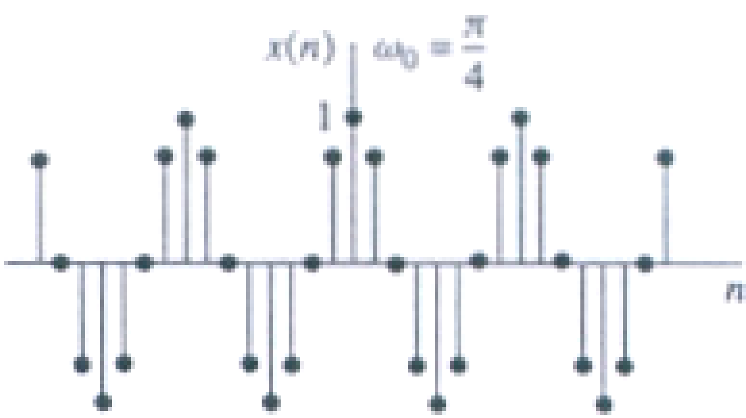
$$x(n) = \cos(\omega_o n)$$











$$x(n) = \cos(\omega n) \qquad \pi < \omega < 2\pi$$

$$x_1(n) = A\cos(\omega_1 n) \qquad \omega_1 = \omega_o$$

$$x_2(n) = A\cos(\omega_2 n) \qquad \omega_2 = 2\pi - \omega_o$$

$$x_2(n) = A\cos(\omega_2 n) = A\cos(2\pi - \omega_o)n$$

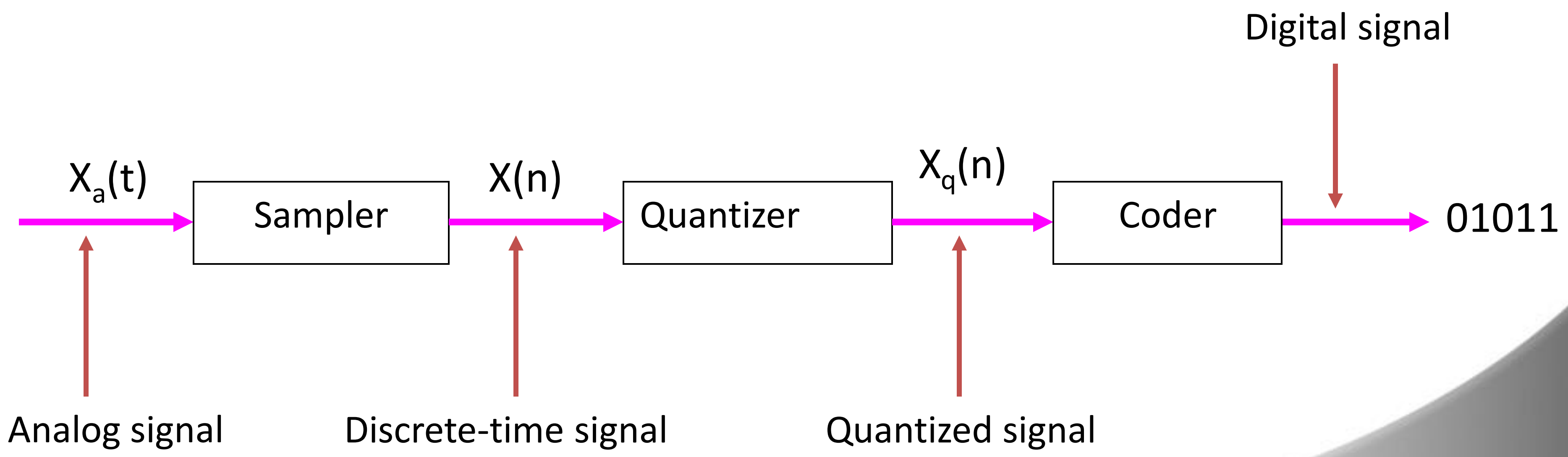
$$= A\cos(2\pi n - \omega_o n) = A\cos(-\omega_o n)$$

$$= A\cos(\omega_o n) = x_1(n)$$

ω_2 adalah alias dari ω_1

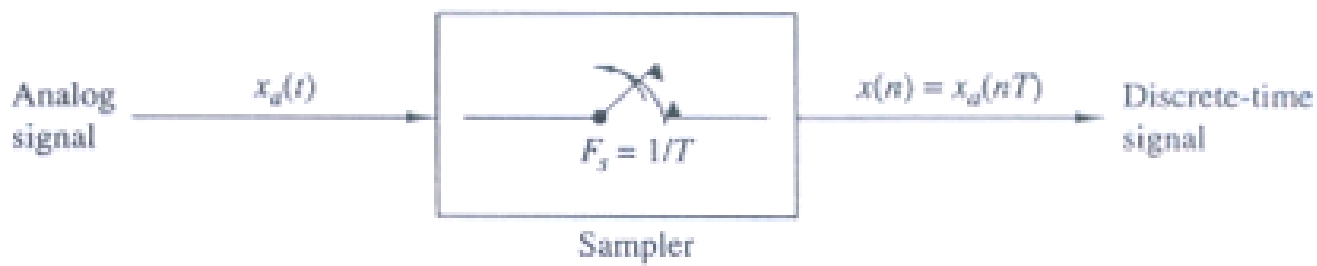
ANALOG TO DIGITAL CONVERSION

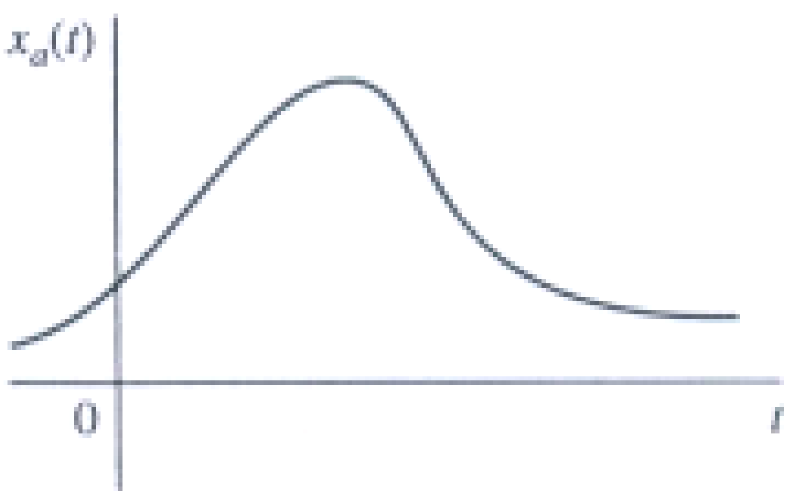
- Sampling (pencuplikan)
- Quantization (kuantisasi)
- Coding (pengkodean)

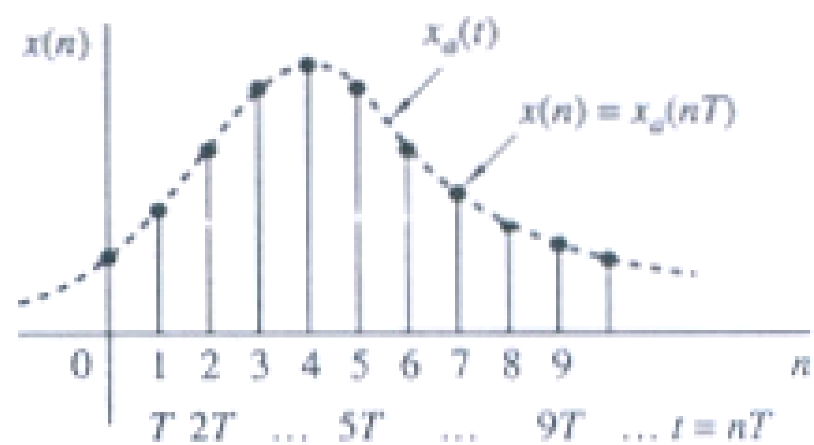


➢ Sampling (pencuplikan)

- Sinyal waktu kontinu → sinyal waktu diskrit
- T = sampling interval
- F_s = sampling rate (sampel/detik)







$$x_a(t) = A\cos(2\pi F t + \theta)$$

$$x_a(nT) = A\cos(2\pi F nT + \theta)$$

$$= A\cos\left(\frac{2\pi nF}{F_s} + \theta\right)$$

$$x(n) = A\cos(2\pi\, f\, n + \theta) \quad \rightarrow \quad f = \frac{F}{F_s}$$

$$f_{\max} = \frac{1}{2} \quad \rightarrow \quad F_{\max} = \frac{F_s}{2} = \frac{1}{2T}$$

$$F > \frac{F_s}{2} \quad \rightarrow \quad ?$$

$$x_1(t) = \cos[2\pi(10)t] \quad \rightarrow F_1 = 10\,Hz$$

$$x_2(t) = \cos[2\pi(50)t] \quad \rightarrow F_2 = 50\,Hz$$

$$F_s = 40\,Hz$$

$$x_1(n) = \cos[2\pi\left(\frac{10}{40}\right)n] = \cos(\frac{\pi}{2}n)$$

$$x_2(n) = \cos[2\pi\left(\frac{50}{40}\right)n] = \cos(\frac{5\pi}{2}n)$$

$$= \cos(2\pi + \frac{\pi}{2})n = \cos(2\pi n + \frac{\pi}{2}n) = \cos(\frac{\pi}{2}n) = x_1(n)$$

$x_2(n)$ identik dengan $x_1(n)$ → F_2 (50 Hz) = alias dari F_1(10 Hz)

90 Hz, 130 Hz, ... juga alias 10 Hz

$$x_a(t) = A\cos(2\pi F_o t + \theta)$$

$$x(n) = A\cos(2\pi f_o n + \theta)$$

$$x_a(t) = A\cos(2\pi F_k t + \theta)$$

$$F_k = F_o + kF_s \quad k = \pm 1, \pm 2, \cdots$$

$$x(n) = x_a(nT) = A\cos(2\pi F_k nT + \theta)$$

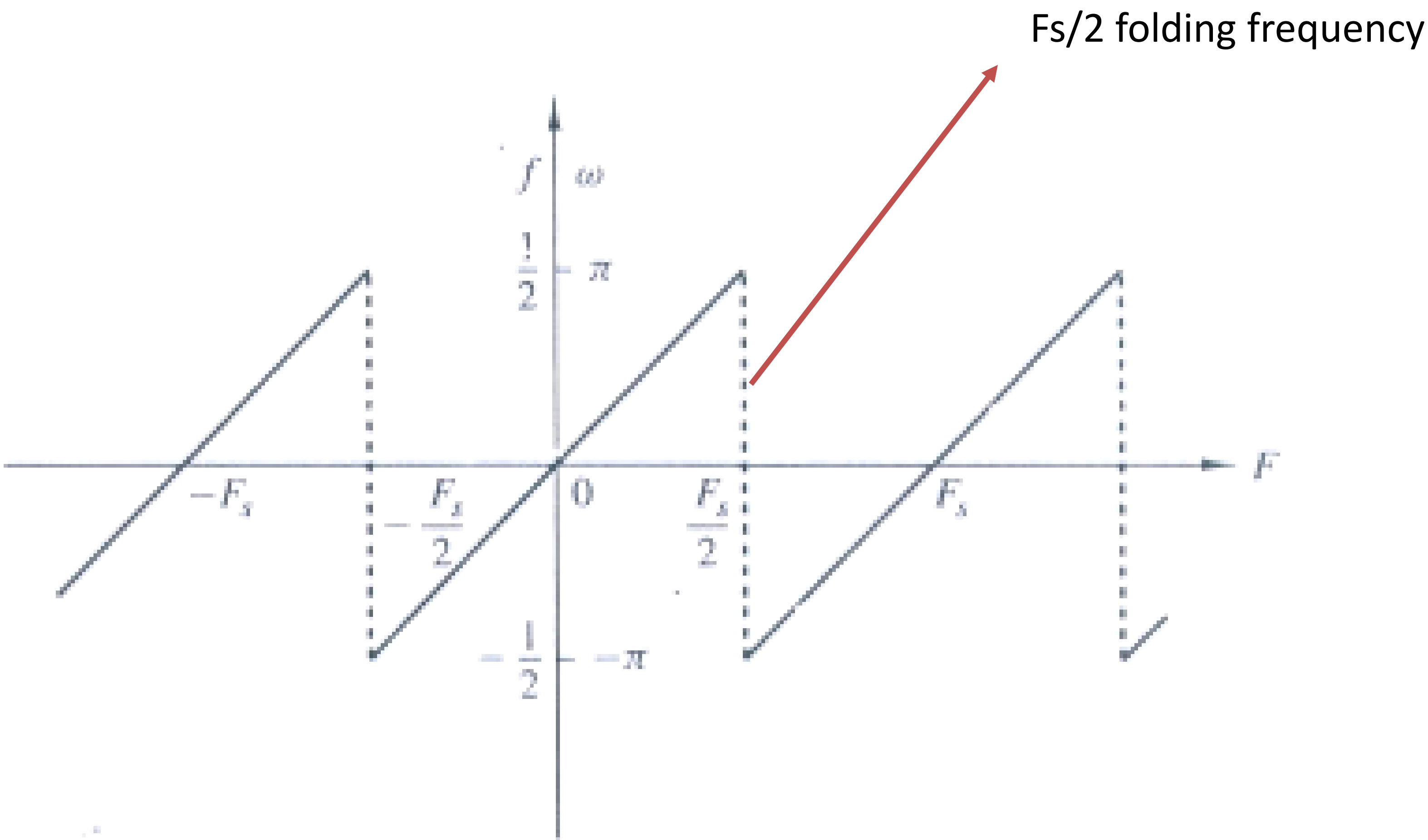
$$x(n) = A\cos\left(2\pi \frac{F_o + kF_s}{F_s} n + \theta\right)$$

$$x(n) = A\cos(2\pi f_o n + 2\pi k + \theta)$$

$$x(n) = A\cos(2\pi f_o n + \theta)$$

Alias dari F_o

Hubungan antara f dan F



Contoh 1:

Diketahui sebuah sinyal analog $x(t) = 3 \cos 100\pi t$

a) Tentukan F_s minimum

b) Bila F_s = 200 Hz, tentukan x(n)

c) Bila F_s = 75 Hz, tentukan x(n)

d) Berapa $0 < F < F_s/2$ yang menghasilkan x(n) sama dengan c)

Jawab:

a) F = 50 Hz → F_s minimum = 100 Hz

b) $$x(n) = 3\cos\frac{100\pi}{200}n = 3\cos\frac{\pi}{2}n$$

c) $$x(n) = 3\cos\frac{100\pi}{75}n = 3\cos\frac{4\pi}{3}n$$

$$= 3\cos(2\pi - \frac{2\pi}{3})n = 3\cos(\frac{2\pi}{3})n$$

d) $$x(n) = 3\cos(\frac{2\pi}{3})n = 3\cos(2\pi\frac{1}{3})n \longrightarrow f = \frac{1}{3}$$

$$f = \frac{F_o}{F_s} \longrightarrow F_o = f\,F_s = \frac{1}{3}(75) = 25\,Hz$$

$$F_k = F_o + kF_s = 25 + k(75) \quad k = \pm 1, \pm 2, \cdots$$

$$0 < F < \frac{F_s}{2} = \frac{75}{2} = 37{,}5 \longrightarrow F = F_o = 25\,Hz$$

Teori Sampling

$$x_a = \sum_{i=1}^{N} A_i \cos(2\pi F_i t + \theta)$$

- **Suara pembicaraan → f_i < 3 kHz**
- **Sinyal televisi → f_i < 5 MHz**
- **F_{maks} = B**
- **F_s = sampling rate = ?**

$$-\frac{1}{2} \leq f = \frac{F}{F_S} \leq \frac{1}{2} \quad \longrightarrow \quad -\frac{F_S}{2} \leq F \leq \frac{F_S}{2}$$

$$2F_{maks} = 2B = F_N \quad \longrightarrow$$ **Frekuensi Nyquist**

Contoh 2:

Diketahui sebuah sinyal analog

$$x(t) = 3\cos(2000\pi t) + 5\sin(6000\pi t) + 10\cos(12000\pi t)$$

a) Tentukan frekuensi Nyquistnya

b) Bila F_s = 5000 Hz, tentukan x(n)

c) Tentukan x(t) dari x(n) pada b) bila proses D/A C nya sempurna

Jawab:

a) $F_1 = 1 kHz \quad F_2 = 3 kHz \quad F_3 = 6 kHz$

$$B = F_{maks} = 6 kHz \longrightarrow F_N = 2B = 12 kHz$$

b) $F_s = 5 kHz \quad \rightarrow \quad \frac{F_s}{2} = 2{,}5 kHz$

$$x(n) = 3\cos\frac{2000\pi}{5000}n + 5\sin\frac{6000\pi}{5000}n + 10\cos\frac{12000\pi}{5000}n$$

$$= 3\cos(2\pi\frac{1}{5})n + 5\sin(2\pi\frac{3}{5})n + 10\cos(2\pi\frac{6}{5})n$$

$$x(n) = 3\cos[2\pi(\frac{1}{5})n] + 5\sin[2\pi(1-\frac{2}{5})n] + 10\cos[2\pi(1+\frac{1}{5})n]$$

$$x(n) = 3\cos[2\pi(\frac{1}{5})n] + 5\sin[2\pi(-\frac{2}{5})n] + 10\cos[2\pi(\frac{1}{5})n]$$

$$x(n) = 3\cos[2\pi(\frac{1}{5})n] + 5\sin[2\pi(-\frac{2}{5})n] + 10\cos[2\pi(\frac{1}{5})n]$$

$$x(n) = 13\cos[2\pi(\frac{1}{5})n] - 5\sin[2\pi(\frac{2}{5})n]$$

c) $y(t) = 13\cos(2000\pi t) - 5\sin(4000\pi t)$