DIGITAL TO ANALOG TO CONVERSION

Kuantisasi sinyal amplituda kontinu

$$x_q(n) = Q[x(n)] \quad \rightarrow \quad e_q(n) = x_q(n) - x(n)$$

Q = proses kuantisasi (rounding, truncation)

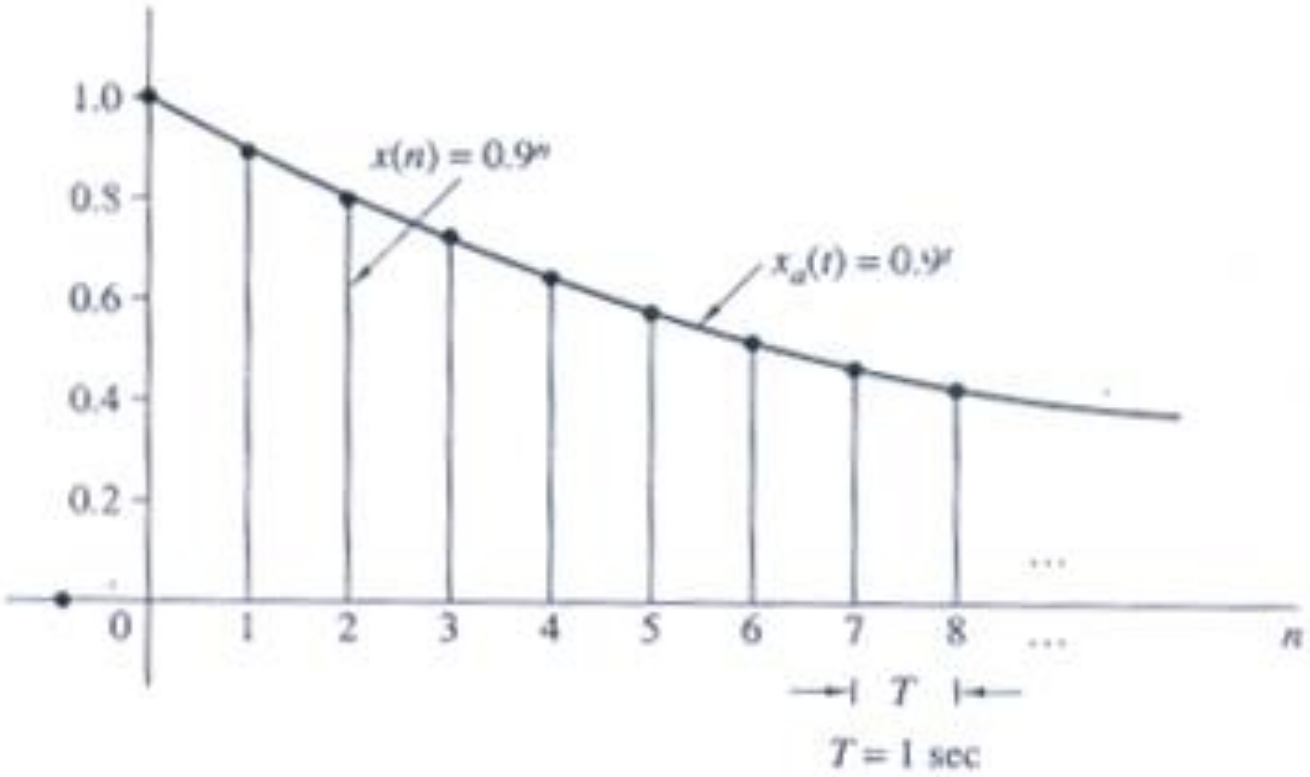
$x_q(n)$ = sinyal hasil kuantisasi

$e_q(n)$ = error kuantisasi

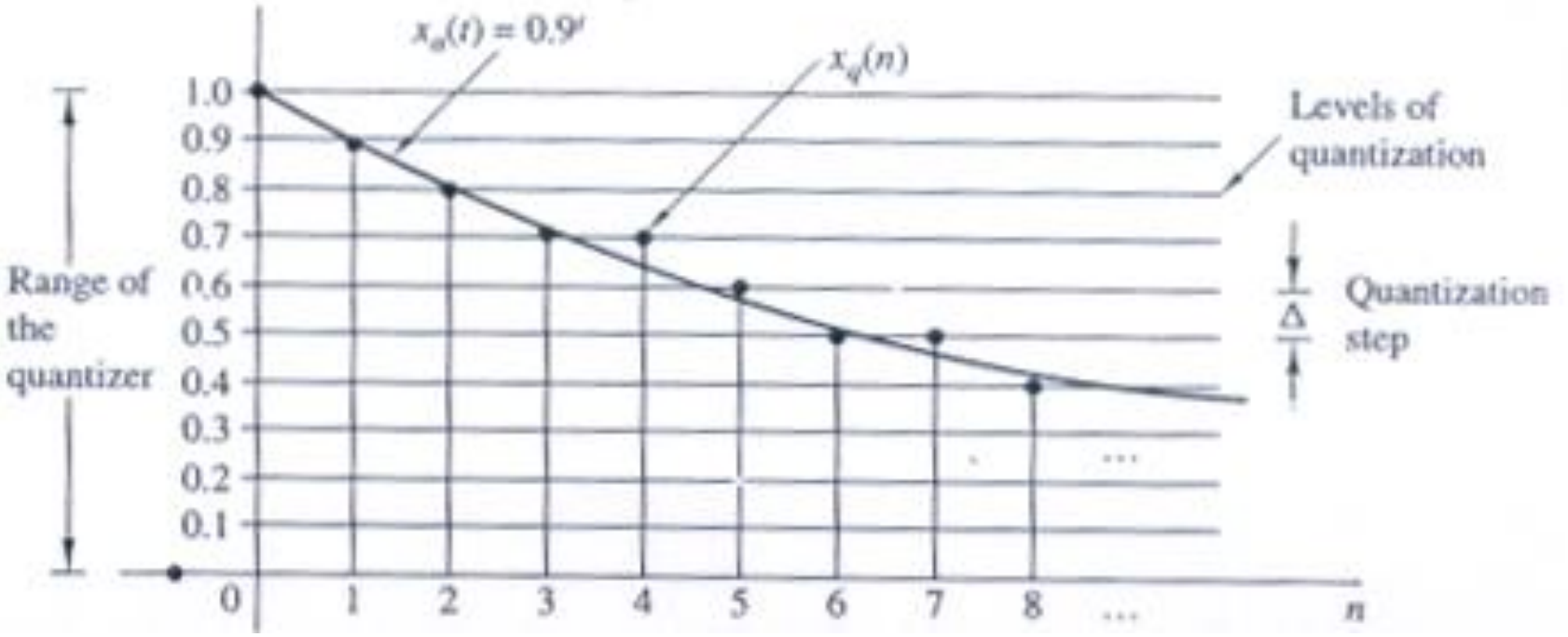
$$x_a(t) = \begin{cases} 0{,}9^t & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$F_S = 1\,Hz \quad \rightarrow \quad T = 1\,s$$

$$x(n) = \begin{cases} 0{,}9^n & n \geq 0 \\ 0 & n < 0 \end{cases}$$



n	x(n)	$x_q(n)$ (Truncation)	$x_q(n)$ (Rounding)	$e_q(n)$ (Rounding)
0	1	1,0	1,0	0,0
1	0.9	0,9	0,9	0,0
2	0.81	0,8	0,8	- 0,01
3	0,729	0,7	0,7	- 0,029
4	0,6561	0,6	0,7	0,0439
5	0,59049	0,5	0,6	0,00951
6	0,5311441	0,5	0,5	- 0,031441
7	0,4782969	0,4	0,5	0,0217071
8	0,43046721	0,4	0,4	- 0,03046721
9	0,387420489	0,3	0,4	0,012579511



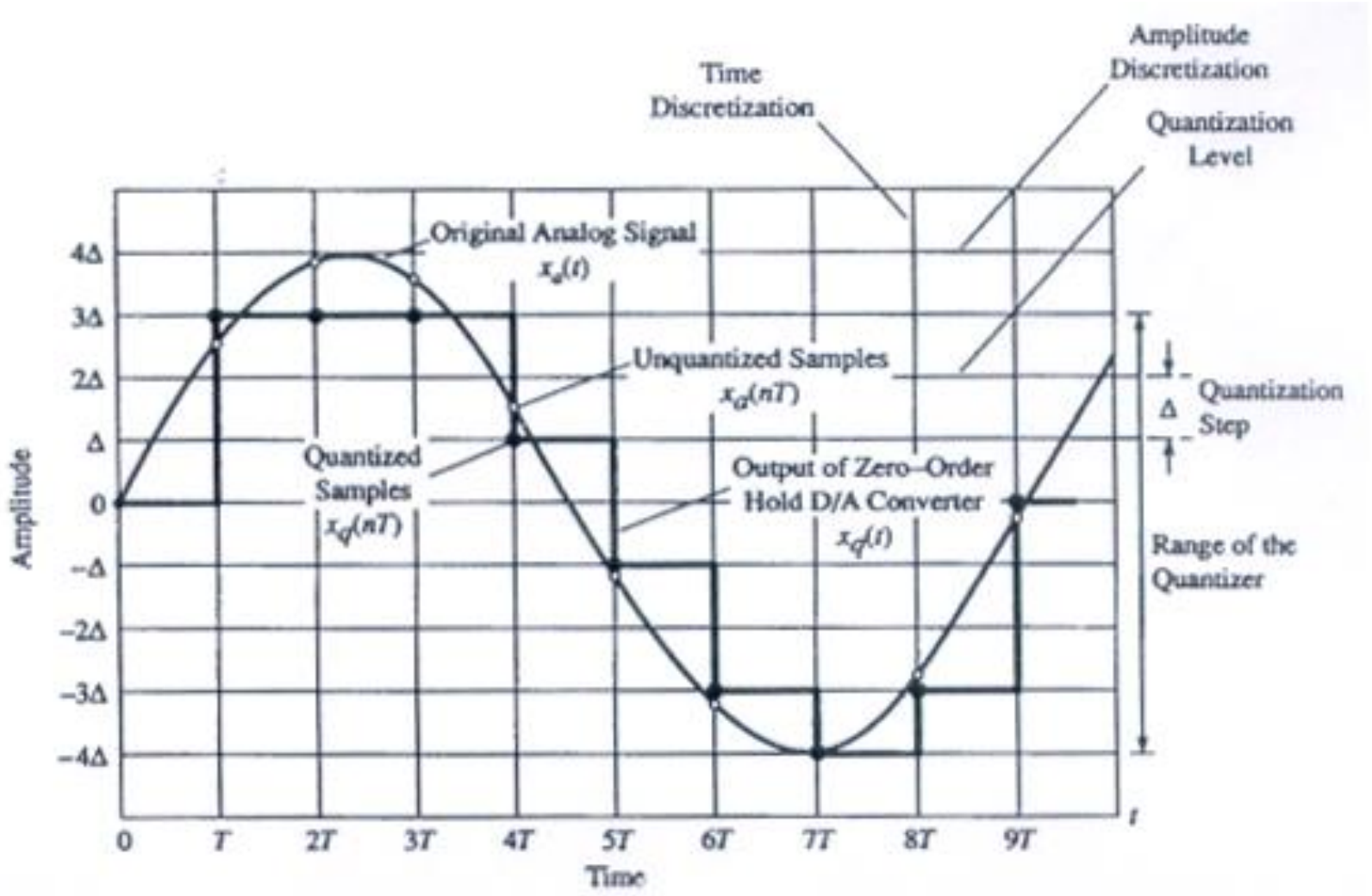
L = level kuantisasi → L = 11

Δ = Quantization step → Δ = 0,1

$$\Delta = \frac{x_{maks} - x_{\min}}{L-1} = \frac{1-0}{11-1} = 0{,}1 \qquad -\frac{\Delta}{2} \le e_q(n) \le \frac{\Delta}{2}$$

Kuantisasi sinyal sinusoidal

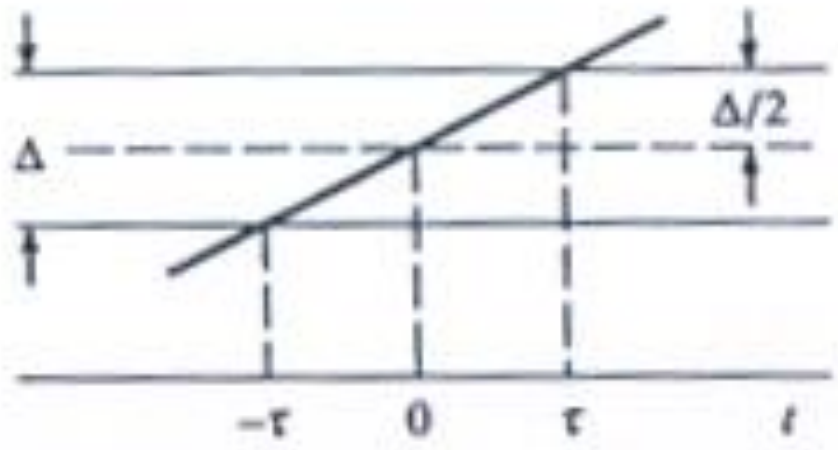
$$x(n) = A\cos(\Omega_0 t)$$

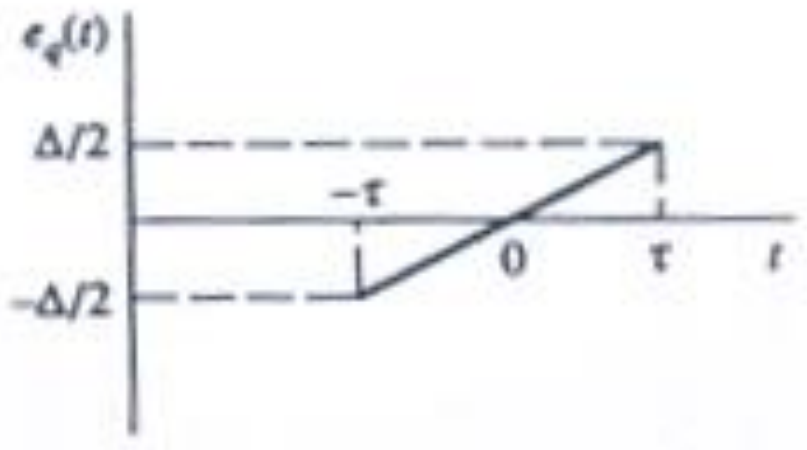


$$F_S \geq 2B \quad \rightarrow \quad e_q(t) = x_a(t) - x_q(t)$$

$x_a(t)$ dianggap linier diantara level-level kuantisasi

τ = waktu selama $x_a(t)$ berada di dalam level kuantisasi





Error power (rms) $\rightarrow$ $$P_q = \frac{1}{2\tau}\int_{-\tau}^{\tau} e_q^2(t)dt = \frac{1}{\tau}\int_{0}^{\tau} e_q^2(t)dt$$

$$e_q(t) = \frac{\Delta}{2\tau} t \quad \rightarrow \quad P_q = \frac{1}{\tau} \int_0^{\tau} \left(\frac{\Delta}{2\tau} \right)^2 t^2 dt = \frac{\Delta^2}{2}$$

b = jumlah bit ➔ **$L = 2^b + 1$**

X_{maks}-x_{min} = 2A

$$\Delta = \frac{2A}{2^b} \quad \rightarrow \quad P_q = \frac{A^2}{3(2^{2b})}$$

$$P_x = \frac{1}{T_p} \int_0^{T_p} (A \cos \Omega_o t)^2 dt = \frac{A^2}{2}$$

Signal-to-quantization ratio ⟹ $$SQNR = \frac{P_x}{P_q} = \frac{3}{2}(2^{2b})$$

$$SQNR(dB) = 10\log SQNR = 1{,}76 + 6{,}02\,b$$

- Word length (jumlah bit) ditambah satu
- Level kuantisasi menjadi dua kali lipat
- SQNR bertambah 6 dB

Contoh :

- Compact disk player
- Sampling frequency 44,1 kHz
- 16-bit sample resolution
- SQNR =96 dB

Coding of Quantized Samples

- Level kuantisasi L → L bilangan biner yang berbeda
- Word lengh b → 2^b bilangan biner berbeda
- $2^b \geq L$ → $b \geq {}^2\log L$
- L = 11 → b = 4 bits

Contoh 4:

Diketahui sinyal waktu diskrit : $x(n) = 6{,}35\cos(\frac{\pi}{10})n$

Tentukan jumlah bit yang diperlukan oleh A/D converter agar resolusinya :

a) $\Delta = 0{,}1$

b) $\Delta = 0{,}02$

Jawab:

a) x(n) maksimum pada saat : $\cos(\frac{\pi}{10})n = 1 \quad \rightarrow \quad n = 0$

x(n) minimum pada saat : $\cos(\frac{\pi}{10})n = -1 \quad \rightarrow \quad n = 10$

$$\Delta = \frac{x_{maks} - x_{\min}}{L-1} \quad \rightarrow \quad L = \frac{x_{maks} - x_{\min}}{\Delta} + 1$$

$$\Delta = 0{,}1 \quad \rightarrow \quad L = \frac{[6{,}35(1) - 6{,}35(-1)]}{0{,}1} + 1 = 128$$

$$2^b \geq 128 \quad \rightarrow \quad b = 7\ bit$$

b) $$\Delta = 0{,}02 \quad \rightarrow \quad L = \frac{[6{,}35(1) - 6{,}35(-1)]}{0{,}02} + 1 = 636$$

$$2^b \geq 636 \quad \rightarrow \quad b = 10\ bit$$

Contoh 5:

Diketahui sinyal seismik analog dengan dynamic range sebesar 1 Volt. Bila sinyal analog ini dicuplik dengan frekuensi sebesar 20 sample/s menggunakan 8-bit A/D converter,

Tentukan :

a) Bit rate (bps)

b) Resolusi

c) Frekuensi sinyal maksimum yang ada pada digital seismic signal

Jawab:

a) $$bps = \frac{8\,bit}{sample}\frac{20\,sample}{s} = 160\,bit/s$$

Dynamic range = $x_{maks} - x_{min}$

b) $$\Delta = \frac{dynamic\ range}{L-1} = \frac{1000\ mV}{2^8 - 1} = 7{,}875\ mV$$

c) $$F_{maks} = \frac{F_S}{2} = \frac{20}{2} = 10\ Hz$$

Contoh 6:

Suatu jaringan komunikasi digital akan digunakan untuk mentransmisikan sinyal analog :

$$x(t) = 3\cos(600\pi t) + 2\cos(1800\pi t)$$

Jaringan ini beroperasi pada 10000 bit/s dan setiap sampel dikuantisasi menjadi 1024 level tegangan yang berbeda.

a) Tentukan frekuensi pencuplikan dan frekuensi folding

b) Tentukan frekuensi Nyquist dari sinyal analog x(t)

c) Tentukan frekuensi-frekuensi pada sinyal waktu diskrit x(n)

Jawab:

a) $1024 = 2^b \quad \rightarrow \quad b = 10\ bit$

$$F_S = \frac{bps}{b} = \frac{10000}{10} = 1000Hz$$

$$F_D = \frac{F_S}{2} = 500Hz$$

b) $x_a(t) = 3\cos(2\pi 300t) + 2\cos(2\pi 900t)$

$F_1 = 300Hz \qquad F_2 = 900Hz$

$F_N = 2F_{maks} = 2F_2 = 2(900) = 1800\ Hz$

c) $$x(n) = 3\cos(2\pi \frac{300}{1000} n) + 2\cos(2\pi \frac{900}{1000} n)$$

$$= 3\cos[2\pi(0{,}3)n] + 2\cos[2\pi(0{,}9)n]$$

$$= 3\cos[2\pi(0{,}3)n] + 2\cos[2\pi(1-0{,}1)n)]$$

$$= 3\cos[2\pi(0{,}3)n] + 2\cos[2\pi(0{,}1)n)]$$

$$f_1 = 0{,}3 \quad \rightarrow \quad F_1 = f_1 F_S = 0{,}3(1000) = 300Hz$$

$$f_2 = 0{,}1 \quad \rightarrow \quad F_2 = f_2 F_S = 0{,}1(1000) = 100Hz$$