PENGOLAHAN SINYAL DIGITAL

Modul 4.

Transformasi Z

PROGRAM SARJANA FAKULTAS TEKNIK
(Undergraduate Program of Engineering School)

Telkom University

Content

- Overview TZ untuk fungsi eksponensial kausal dan anti kausal, ROC, Zero Pole, TZ fungsi impuls, TZ fungsi sinusoidal
- Overview ITZ : Pecahan Parsial dan Integrasi Kontur, manipulasi ITZ berdasarkan propertynya, ROCnya (kausal dan anti kausal), fungsinya. contoh : ITZ fungsi logaritma f(z) dan TZ fungsi x(n)/n.

Latar Belakang

"Domains of representation "

- Domain-n (discrete time) :
  Sequence, impulse response, persamaan beda
- Domain- ω :
  Freq. response, spectral representation
- Domain-z :
  Operator, dan pole-zero

Apabila suatu kasus sulit dipecahkan pada suatu domain tertentu, maka transformasi ke domain yang lain akan mudah menyelesaikannya.

Content

- Transformasi-Z Langsung
- Sifat-sifat Transformasi-Z
- Transformasi-Z Rasional
- InversTransformasi-Z
- Transformasi-Z Satu Sisi

TRANSFORMASI-Z LANGSUNG

- **Definisi :**

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

- **Contoh 1:**

a. $x_1(n) = \{1, 2, 5, 7, 0, 1\}$

$$X_1(z) = 1 + 2z^{-1} + 5z^{-2} + 7z^{-3} + z^{-5}$$

b. $x_2(n) = \{1, 2, \underset{\uparrow}{5}, 7, 0, 1\}$

$$X_2(z) = z^{2} + 2z^{1} + 5 + 7z^{-1} + z^{-3}$$

Contoh 2:

Tentukan transformasi Z dari beberapa sinyal di bawah ini:

a. $x_1(n) = \delta(n)$

b. $x_2(n) = \delta(n-k),\ k > 0$

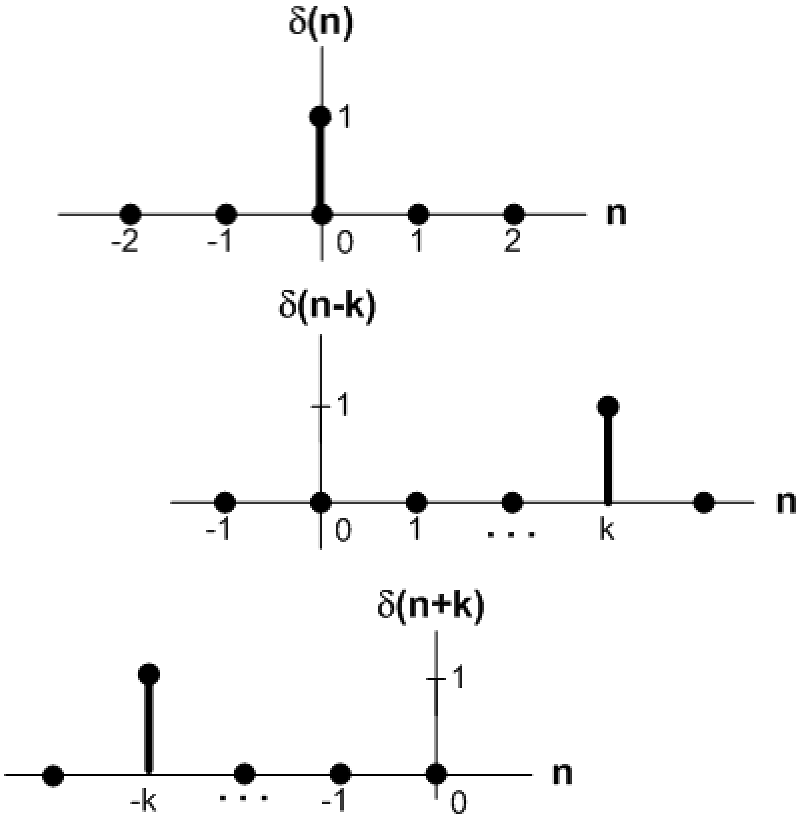
c. $x_3(n) = \delta(n+k),\ k > 0$

Jawab:

a. $$X_1(z) = \sum_{n=-\infty}^{\infty} \delta(n) z^{-n} = 1 \,.\, z^0 = 1$$

b. $$X_2(z) = \sum_{n=-\infty}^{\infty} \delta(n-k) z^{-n} = z^{-k}$$

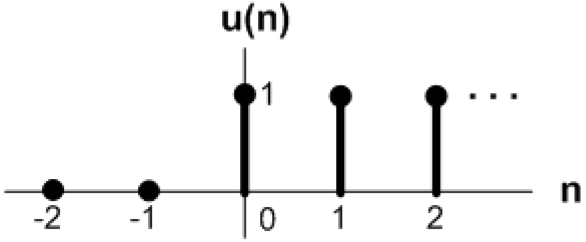
c. $$X_3(z) = \sum_{n=-\infty}^{\infty} \delta(n+k) z^{-n} = z^{k}$$



Contoh 3:

Tentukan transformasi Z dari sinyal $x(n) = u(n)$

Jawab:



$$X(z) = \sum_{n=0}^{\infty} u(n)\, z^{-n} = 1 + z^{-1} + z^{-2} + \ldots$$

$$= \frac{1}{1 - z^{-1}} \text{, dimana } \left|z^{-1}\right| < 1 \rightarrow ROC: |z| > 1$$

$$\therefore \quad x(n) = u(n) \quad \rightarrow \quad X(z) = \frac{1}{1 - z^{-1}} \quad , ROC: |z| > 1$$

- ## Contoh 4:

Tentukan transformasi Z dari sinyal $x(n) = \alpha^n u(n)$

Jawab:

$$X(z) = \sum_{n=0}^{\infty} \alpha^n u(n) z^{-n} = \sum_{n=0}^{\infty} \left(\alpha z^{-1}\right)^n$$

$$= \sum_{n=0}^{\infty} (A)^n = 1 + A + A^2 + A^3 + \ldots = \frac{1}{1-A}$$

$$= \frac{1}{1-\alpha z^{-1}} \text{, dimana } \left|\alpha z^{-1}\right| < 1 \rightarrow ROC : |z| > \alpha$$

$$\therefore \quad x(n) = \alpha^n u(n) \quad \rightarrow \quad X(z) = \frac{1}{1-\alpha z^{-1}} \quad , ROC : |z| > \alpha$$

TABEL FUNGSI DASAR TZ

Sequence	z-Transform	Region of Convergence
$\delta(n)$	1	all z
$\alpha^n u(n)$	$\frac{1}{1-\alpha z^{-1}}$	$\lvert z\rvert > \lvert\alpha\rvert$
$-\alpha^n u(-n-1)$	$\frac{1}{1-\alpha z^{-1}}$	$\lvert z\rvert < \lvert\alpha\rvert$
$n\alpha^n u(n)$	$\frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}$	$\lvert z\rvert > \lvert\alpha\rvert$
$-n\alpha^n u(-n-1)$	$\frac{\alpha z^{-1}}{(1-\alpha z^{-1})^2}$	$\lvert z\rvert < \lvert\alpha\rvert$
$\cos(n\omega_0)u(n)$	$\frac{1-(\cos\omega_0)z^{-1}}{1-2(\cos\omega_0)z^{-1}+z^{-2}}$	$\lvert z\rvert > 1$
$\sin(n\omega_0)u(n)$	$\frac{(\sin\omega_0)z^{-1}}{1-2(\cos\omega_0)z^{-1}+z^{-2}}$	$\lvert z\rvert > 1$

SIFAT-SIFAT (PROPERTY) TZ

Property	Sequence	z-Transform	Region of Convergence
Linearity	$ax(n) + by(n)$	$aX(z) + bY(z)$	Contains $R_x \cap R_y$
Shift	$x(n - n_0)$	$z^{-n_0}X(z)$	R_x
Time reversal	$x(-n)$	$X(z^{-1})$	$1/R_x$
Exponentiation	$\alpha^n x(n)$	$X(\alpha^{-1}z)$	$\lvert\alpha\rvert R_x$
Convolution	$x(n) * y(n)$	$X(z)Y(z)$	Contains $R_x \cap R_y$
Conjugation	$x^*(n)$	$X^*(z^*)$	R_x
Derivative	$nx(n)$	$-z\dfrac{dX(z)}{dz}$	R_x

SIFAT-SIFAT TRANSFORMASI-Z

- **Linieritas**

$$x(n) = a\,x_1(n) + b\,x_2(n) \quad \rightarrow \quad X(z) = a\,X_1(z) + b\,X_2(z)$$

- **Contoh 5:**

Tentukan transformasi Z dari sinyal $x(n) = \left[3(2)^n - 4(3)^n\right]u(n)$

Jawab:

$$x_1(n) = (2)^n u(n) \rightarrow X_1(z) = \frac{1}{1-2z^{-1}} \quad , ROC: |z| > 2$$

$$x_2(n) = (3)^n u(n) \rightarrow X_2(z) = \frac{1}{1-3z^{-1}} \quad , ROC: |z| > 3$$

$$x(n) = \left[3(2)^n - 4(3)^n\right]u(n) \rightarrow X(Z) = \frac{3}{1-2z^{-1}} - \frac{4}{1-3z^{-1}} = \frac{-1-z^{-1}}{1-5z^{-1}+6z^{-2}}$$

$$ROC: |z| > 2 \cap |z| > 3 \rightarrow ROC: |z| > 3$$

SIFAT-SIFAT TRANSFORMASI-Z

- **Pergeseran**

$$x(n-n_0) \quad \rightarrow \quad z^{-n_0} X(Z)$$

- **Contoh 5:**

Tentukan transformasi Z dari sinyal $x(n)=u(n-3)$

Jawab:

$$x_1(n)=u(n)\rightarrow X_1(Z)=\frac{1}{1-z^{-1}}, ROC: R_x=|z|>1$$

$$\therefore \; x(n)=u(n-3)\rightarrow X(Z)=z^{-3}X_1(Z)=\frac{z^{-3}}{1-z^{-1}}, \; ROC: R_x=|z|>1$$

SIFAT-SIFAT TRANSFORMASI-Z

- **Time Reversal**

$$x(-n) \quad \rightarrow \quad X(z^{-1})$$

- **Contoh 6:**

Tentukan transformasi Z dari sinyal $x(n) = u(-n)$

Jawab:

$$x_1(n) = u(n) \rightarrow X_1(Z) = \frac{1}{1-z^{-1}}, ROC : R_x = |z| > 1$$

$$\therefore \ x(n) = u(-n) \rightarrow X(z) = \frac{1}{1-(z^{-1})^{-1}} = \frac{1}{1-z}, \ ROC : \ {}^{1}\!/\!{}_{R_x} = |z| < 1$$

SIFAT-SIFAT TRANSFORMASI-Z

- **Diferensiasi dalam domain z**

$$nx(n) \quad \rightarrow \quad -z\frac{dX(z)}{dz}$$

- **Contoh 7:**

Tentukan transformasi Z dari sinyal $x(n) = n\,a^n u(n)$

Jawab:

$$x_1(n) = a^n u(n) \quad \rightarrow \quad X_1(z) = \frac{1}{1-az^{-1}},\ ROC: R_x = |z| > a$$

$$\therefore\ x(n) = n\,a^n u(n) \quad \rightarrow \quad X(z) = -z\frac{dX_1(z)}{dz} = -z\frac{d}{dz}\left(\frac{1}{1-az^{-1}}\right)$$

$$= (-z)\frac{-az^{-2}}{\left(1-az^{-1}\right)^2} = \frac{az^{-1}}{\left(1-az^{-1}\right)^2}$$

$$\therefore\ n\,a^n u(n) \quad \rightarrow \quad \frac{az^{-1}}{\left(1-az^{-1}\right)^2}$$

SIFAT-SIFAT TRANSFORMASI-Z

- **Konvolusi antara dua sinyal**

$$x(n) = x_1(n) * x_2(n) \quad \rightarrow \quad X(z) = X_1(z)X_2(z)$$

- **Contoh 8:**

Tentukan konvolusi antara $x_1(n)$ dan $x_2(n)$ dengan :

$$x_1(n) = \{1, -2, 1\} \qquad x_2(n) = \begin{cases} 1, & 0 \le n \le 5 \\ 0, & lainnya \end{cases}$$

Jawab:

$$X_1(z) = 1 - 2z^{-1} + z^{-2} \qquad X_2(z) = 1 + z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5}$$

$$X(z) = X_1(z)X_2(z) = (1 - 2z^{-1} + z^{-2})(1 + z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5})$$

$$X(z) = X_1(z)X_2(z) = 1 - z^{-1} - z^{-6} + z^{-7}$$

$$\therefore\ x(n) = x_1(n) * x_2(n) = \{1, -1, 0, 0, 0, 0, -1, 1\}$$

TRANSFORMASI Z RASIONAL

Pole dan Zero

Pole : harga-harga z = p_i yang menyebabkan X(z) = ∞

Zero : harga-harga z = z_i yang menyebabkan X(z) = 0

Fungsi Rasional

$$X(z)=\frac{N(z)}{D(z)}=\frac{b_o+b_1z^{-1}+\cdots+b_Mz^{-M}}{a_o+a_1z^{-1}+\cdots+a_Nz^{-N}}=\frac{\sum_{k=0}^{M}b_kz^{-k}}{\sum_{k=0}^{N}a_kz^{-k}}$$

$$a_o\neq 0\;\; b_o\neq 0\quad\rightarrow\quad X(z)=\frac{N(z)}{D(z)}=\frac{b_oz^{-M}}{a_oz^{-N}}\;\frac{z^M+\left(\frac{b_1}{b_o}\right)z^{M-1}+\cdots+\left(\frac{b_M}{b_o}\right)}{Z^N+\left(\frac{a_1}{a_o}\right)z^{N-1}+\cdots+\left(\frac{a_N}{a_o}\right)}$$

$$a_o \neq 0 \;\; b_o \neq 0 \quad \rightarrow \quad X(z) = \frac{N(z)}{D(z)} = \frac{b_o z^{-M}}{a_o z^{-N}} \; \frac{z^M + \left(\frac{b_1}{b_o}\right) z^{M-1} + \cdots + \left(\frac{b_M}{b_o}\right)}{Z^N + \left(\frac{a_1}{a_o}\right) z^{N-1} + \cdots + \left(\frac{a_N}{a_o}\right)}$$

- **N(z) dan D(z) polinom**

$$X(z) = \frac{N(z)}{D(z)} = \frac{b_o}{a_o} z^{N-M} \; \frac{(z - z_1)(z - z_2)\cdots(z - z_M)}{(z - p_1)(z - p_2)\cdots(z - p_M)}$$

$$X(z) = G\, z^{N-M} \; \frac{\prod_{k=1}^{M} (z - z_k)}{\prod_{k=1}^{N} (z - p_k)}$$

- ## Contoh 9:

Tentukan pole dan zero dari $X(z) = \dfrac{2 - 1{,}5z^{-1}}{1 - 1{,}5z^{-1} + 0{,}5z^{-2}}$

Jawab:

$$X(z) = 2\,\frac{z^{-1}}{z^{-2}}\,\frac{z - 0{,}75}{z^2 - 1{,}5z + 0{,}5}$$

$$= 2z\,\frac{z - 0{,}75}{(z-1)(z-0{,}5)} = \frac{2z(z - 0{,}75)}{(z-1)(z-0{,}5)}$$

$$\therefore \quad Zero: \; z_1 = 0 \quad z_2 = 0{,}75$$

$$Pole: \; p_1 = 1 \quad p_2 = 0{,}5$$

- **Contoh 10:**

Tentukan pole dan zero dari $$X(z) = \frac{1+z^{-1}}{1-z^{-1}+0{,}5z^{-2}}$$

Jawab:

$$X(z) = \frac{z(z+1)}{z^2 - z + 0{,}5}$$

$$= \frac{z(z+1)}{[z-(0{,}5+j0{,}5)][z-(0{,}5-j0{,}5)]}$$

$$\therefore \; Zero: \; z_1 = 0 \quad z_2 = 1$$

$$Pole: \; p_1 = 0{,}5 + j0{,}5 \qquad p_2 = 0{,}5 - j0{,}5 \rightarrow p_1 = p_2^{\,*}$$

INVERS TRANSFORMASI -Z

- **Definisi Invers Transformasi-Z**

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

→

$$x(n) = \frac{1}{2\pi j} \oint X(z) z^{n-1} dz$$

Teorema residu Cauchy :

$$\frac{1}{2\pi j} \oint_C \frac{f(z)}{(z-z_o)^k} dz = \begin{cases} \frac{1}{(k-1)!} \left. \frac{d^{k-1} f(z)}{dz^{k-1}} \right|_{z=z_o}, & bila\ z_o\ di\ dalam\ C \\ 0, & bila\ z_o\ di\ luar\ C \end{cases}$$

Ekspansi deret dalam z dan z^{-1}

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

Contoh 11:

Tentukan invers transformasi-z dari $X(z) = \dfrac{1}{1 - \frac{3}{2} z^{-1} + \frac{1}{2} z^{-2}}$

Jawab:

$$X(z) = 1 + \frac{3}{2} z^{-1} + \frac{7}{4} z^{-2} + \frac{15}{8} z^{-3} + \frac{31}{16} z^{-4} \cdots$$

$$\therefore \; x(n) = \left\{ \underset{\uparrow}{1}, \frac{3}{2}, \frac{7}{4}, \frac{15}{8}, \frac{31}{16}, \cdots \right\}$$

- **Ekspansi fraksi-parsial dan tabel transformasi-z**

$$X(z) = \alpha_1 X_1(z) + \alpha_2 X_2(z) + \cdots + \alpha_K X_K(z)$$

$$x(n) = \alpha_1 x_1(n) + \alpha_2 x_2(n) + \cdots + \alpha_K x_K(n)$$

- **Contoh 12:**

Tentukan invers transformasi-z dari $X(z) = \dfrac{1}{1 - 1{,}5z^{-1} + 0{,}5z^{-2}}$

Jawab:

$$X(z) = \frac{z^2}{z^2 - 1{,}5z + 0{,}5} = \frac{z^2}{(z-1)(z-0{,}5)}$$

$$\frac{X(z)}{z} = \frac{z}{z^2 - 1{,}5z + 0{,}5} = \frac{A_1}{(z-1)} + \frac{A_2}{(z-0{,}5)}$$

$$\frac{X(z)}{z}=\frac{z}{z^2-1{,}5z+0{,}5}=\frac{A_1}{(z-1)}+\frac{A_2}{(z-0{,}5)}$$

$$=\frac{A_1(z-0{,}5)+A_2(z-1)}{(z-1)(z-0{,}5)}=\frac{(A_1+A_2)z-(0{,}5A_1+A_2)}{z^2-1{,}5z+0{,}5}$$

$$\frac{X(z)}{z}=\frac{z}{z^2-1{,}5z+0{,}5}=\frac{(A_1+A_2)z-(0{,}5A_1+A_2)}{z^2-1{,}5z+0{,}5}$$

$$A_1+A_2=1 \quad 0{,}5A_1+A_2=0 \quad \rightarrow \quad A_2=-0{,}5A_1$$

$$A_1-0{,}5A_1=0{,}5A_1=1 \quad \rightarrow \quad A_1=2 \quad \rightarrow \quad A_2=-1$$

$$\frac{X(z)}{z}=\frac{2}{(z-1)}-\frac{1}{(z-0{,}5)} \Longrightarrow X(z)=\frac{2z}{(z-1)}-\frac{z}{(z-0{,}5)}$$

$$X(z)=\frac{2}{(1-z^{-1})}-\frac{1}{(1-0{,}5z^{-1})} \Longrightarrow \therefore\ x(n)=[2-(0{,}5)^n]u(n)$$

- **Pole-pole berbeda semua**

$$\frac{X(z)}{z} = \frac{A_1}{z-p_1} + \cdots + \frac{A_k}{z-p_k} + \cdots + \frac{A_N}{z-p_N}$$

$$\frac{(z-p_k)X(z)}{z} = \frac{(z-p_k)A_1}{z-p_1} + \cdots + A_k + \cdots + \frac{(z-p_k)A_N}{z-p_N}$$

$$\left.\frac{(z-p_k)X(z)}{z}\right|_{z=p_k} = A_k$$

Contoh 13:

Tentukan invers transformasi-z dari $X(z) = \dfrac{\left(3 - 2z^{-1}\right)}{\left(1 + 6z^{-1} + 8z^{-2}\right)}$

Jawab:

$$\frac{X(z)}{z} = \frac{\left(3z - 2\right)}{\left(z^2 + 6z + 8\right)} = \frac{A_1}{z+2} + \frac{A_2}{z+4}$$

$$A_1 = \frac{(z+2)X(z)}{z} = \left.\frac{3z-2}{(z+4)}\right|_{z=-2} = \frac{-8}{2} = -4$$

$$A_2 = \frac{(z+4)X(z)}{z} = \left.\frac{3z-2}{(z+2)}\right|_{z=-4} = \frac{-14}{-2} = 7$$

$$\frac{X(z)}{z} = \frac{-4}{z+2} + \frac{7}{z+4} \Longrightarrow X(z) = \frac{-4}{1+2z^{-1}} + \frac{7}{1+4z^{-1}}$$

$$\therefore\ x(n) = [-4(-2)^n + 7(-4)^n]u(n)$$

- **Ada dua pole yang semua**

$$\frac{X(z)}{z} = \frac{A_1}{z - p_1} + \cdots + \frac{A_{1k}}{(z - p_k)^2} + \frac{A_{2k}}{z - p_k} + \cdots + \frac{A_N}{z - p_N}$$

$$A_{1k} = \left. \frac{(z - p_k)^2 X(z)}{z} \right|_{z = p_k}$$

$$A_{2k} = \frac{d}{dz} \left[\frac{(z - p_k)^2 X(z)}{z} \right]_{z = p_k}$$

- **Contoh 14:**

Tentukan invers transformasi-z dari $X(z)=\dfrac{1}{\left(1+z^{-1}\right)\left(1-z^{-1}\right)^2}$

Jawab:

$$\frac{X(z)}{z}=\frac{z^2}{(z+1)(z-1)^2}=\frac{A_1}{z+1}+\frac{A_2}{(z-1)^2}+\frac{A_3}{(z-1)}$$

$$A_1=\frac{(z+1)X(z)}{z}=\left.\frac{z^2}{(z-1)^2}\right|_{z=-1}=\frac{1}{4}$$

$$A_2=\frac{(z-1)^2X(z)}{z}=\left.\frac{z^2}{(z+1)}\right|_{z=1}=\frac{1}{2}$$

$$A_3 = \frac{d}{dz}\left[\frac{(z-1)^2 X(z)}{z}\right] = \frac{d}{dz}\left[\frac{z^2}{(z+1)}\right]$$

$$= \frac{(2z)(z+1) - (1)(z^2)}{(z+1)^2} = \left.\frac{z^2 + 2z}{(z+1)^2}\right|_{z=1} = \frac{3}{4}$$

$$\frac{X(z)}{z} = \frac{1/4}{z+1} + \frac{1/2}{(z-1)^2} + \frac{3/4}{(z-1)}$$

$$\therefore\ x(n) = \left[\frac{1}{4}(-1)^n + \frac{1}{2}n + \frac{3}{4}\right]u(n)$$

- **Pole kompleks**

$$X(z) = \frac{A_1}{1 - p_1 z^{-1}} + \frac{A_2}{1 - p_2 z^{-1}}$$

$$p_1 = p \quad \rightarrow \quad p_2 = p^*$$

$$A_1 = A \quad \rightarrow \quad A_2 = A^*$$

$$\frac{A}{1 - pz^{-1}} + \frac{A^*}{1 - p^* z^{-1}} = \frac{A - Ap^* z^{-1} + A^* - A^* pz^{-1}}{1 - pz^{-1} - p^* z^{-1} + pp^* z^{-2}}$$

$$\frac{(A + A^*) - (Ap^* + A^* p)z^{-1}}{1 - (p + p^*)z^{-1} + pp^* z^{-2}} = \frac{b_o + b_1 z^{-1}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

$$A + A^* = \mathrm{Re}(A) + j\,\mathrm{Im}(A) + \mathrm{Re}(A) - j\,\mathrm{Im}(A) = 2\,\mathrm{Re}(A)$$

$$b_o = A + A^* = 2\,\mathrm{Re}(A)$$

$$p + p^* = \mathrm{Re}(p) + j\,\mathrm{Im}(p) + \mathrm{Re}(p) - j\,\mathrm{Im}(p) = 2\,\mathrm{Re}(p)$$

$$a_1 = (p + p^*) = -2\,\mathrm{Re}(p)$$

$$pp^* = [\mathrm{Re}(p) + j\,\mathrm{Im}(p)][\mathrm{Re}(p) - j\,\mathrm{Im}(p)]$$

$$= \mathrm{Re}^2(p) + \mathrm{Im}^2(p) = |p|^2 \quad \rightarrow \quad a_2 = pp^* = |p|^2$$

$$Ap^* + A^*p = [\mathrm{Re}(A) + j\,\mathrm{Im}(A)][\mathrm{Re}(p) - j\,\mathrm{Im}(p)]$$

$$+ [\mathrm{Re}(A) - j\,\mathrm{Im}(A)][\mathrm{Re}(p) + j\,\mathrm{Im}(p)]$$

$$= 2\,\mathrm{Re}(A)\,\mathrm{Re}(p) + 2\,\mathrm{Im}(A)\,\mathrm{Im}(p)$$

$$Ap^* = [\mathrm{Re}(A) + j\,\mathrm{Im}(A)][\mathrm{Re}(p) - j\,\mathrm{Im}(p)]$$

$$= [\mathrm{Re}(A)\,\mathrm{Re}(p) + \mathrm{Im}(A)\,\mathrm{Im}(p)] + j\,[\mathrm{Re}(p)\,\mathrm{Im}(A) - \mathrm{Re}(A)\,\mathrm{Im}(p]$$

$$b_1 = -(Ap^* + A^*p) = -2\,\mathrm{Re}(Ap^*)$$

- **Contoh 15:**

Tentukan invers transformasi-z dari $X(z)=\dfrac{1+z^{-1}}{1-z^{-1}+0{,}5z^{-2}}$

Jawab:

$$X(z)=\frac{1+z^{-1}}{1-z^{-1}+0{,}5z^{-2}}=\frac{b_o+b_1z^{-1}}{1+a_1z^{-1}+a_2z^{-2}}$$

$$\begin{array}{lcl} b_o=2\,\mathrm{Re}(A)=1 & \rightarrow & \mathrm{Re}(A)=0{,}5 \\ a_1=-2\,\mathrm{Re}(p)=-1 & \rightarrow & \mathrm{Re}(p)=0{,}5 \\ b_1=2\,\mathrm{Re}(Ap^*)=1 & \rightarrow & \mathrm{Re}(Ap^*)=0{,}5 \\ a_2=\left|p\right|^2=0{,}5 & \rightarrow & \mathrm{Re}^2(p)+\mathrm{Im}^2(p)=0{,}5 \end{array}$$

$$\mathrm{Re}(p) = 0{,}5 \qquad \mathrm{Re}(A) = 0{,}5$$

$$\mathrm{Re}^2(p) + \mathrm{Im}^2(p) = 0{,}25 + \mathrm{Im}^2(p) = 0{,}5$$

$$\mathrm{Im}^2(p) = 0{,}25 \;\rightarrow\; \mathrm{Im}(p) = 0{,}5 \;\rightarrow\; p = 0{,}5 + j\,0{,}5$$

$$Ap^* = [0{,}5 + j\,\mathrm{Im}(A)](0{,}5 - j0{,}5)$$

$$\mathrm{Re}(Ap^*) = 0{,}25 + 0{,}5\,\mathrm{Im}(A) = 0{,}5$$

$$\mathrm{Im}(A) = 0{,}5 \quad \rightarrow \quad A = 0{,}5 + j\,0{,}5$$

$$X(z) = \frac{A}{1 - pz^{-1}} + \frac{A^*}{1 - p^* z^{-1}}$$

$$= \frac{0{,}5 + j\,0{,}5}{1 - (0{,}5 + j\,0{,}5)z^{-1}} + \frac{0{,}5 - j\,0{,}5}{1 - (0{,}5 - j\,0{,}5)z^{-1}}$$

$$X(z) = \frac{0,5 + j\,0,5}{1-(0,5+j\,0,5)z^{-1}} + \frac{0,5 - j\,0,5}{1-(0,5-j\,0,5)z^{-1}}$$

$$0,5 + \mathrm{j}\,0,5 = 0,707\mathrm{e}^{\mathrm{j}45} \qquad 0,5 - \mathrm{j}\,0,5 = 0,707\mathrm{e}^{-\mathrm{j}45}$$

$$\begin{aligned}
x(n) &= (0,5+j\,0,5)(0,707e^{j45})^n + (0,5-j\,0,5)(0,707e^{j45})^n \\
&= (0,5)(0,707)^n(\cos 45n + j\sin 45n) \\
&\quad + j(0,5)(0,707^n)(\cos 45n + j\sin 45n) \\
&\quad + (0,5)(0,707)^n(\cos 45n - j\sin 45n) \\
&\quad - j(0,5)(0,707^n)(\cos 45n - j\sin 45n) \\
&= (0,707)^n \cos 45n - (0,707)^n \sin 45n
\end{aligned}$$