PENGOLAHAN SINYAL DIGITAL

Modul 5.

Sistem Waktu Diskret dan Aplikasi TZ

FAKULTAS PASCA SARJANA
(Graduate School)

Telkom University

Content

- Overview Sistem Waktu Diskrit
- System Properties (Shift Invariance, Kausalitas, Stabilitas) dikaitkan dengan TZ
- Transformasi sistem dari persamaan difference ke respon impuls dan sebaliknya
- Realisasi Sistem dg adder minimal dan delay minimal
- Mencari Respon Steady State
- Struktur : kaskade, paralel

- **Fungsi Sistem dari Sistem LTI**

$$\boxed{y(n)=h(n)*x(n)\ \rightarrow\ Y(z)=H(z)X(z)\ \rightarrow\ H(z)=\frac{Y(z)}{X(z)}}$$

Respon impuls $h(n)\ \rightarrow\ H(z)$ **Fungsi sistem**

Persamaan beda dari sistem LTI :

$$y(n)=-\sum_{k=1}^{N}a_k y(n-k)+\sum_{k=0}^{M}b_k x(n-k)$$

$$Y(z)=-\sum_{k=1}^{N}a_k Y(z)z^{-k}+\sum_{k=0}^{M}b_k X(z)z^{-k}$$

$$Y(z) = -\sum_{k=1}^{N} a_k Y(z) z^{-k} + \sum_{k=0}^{M} b_k X(z) z^{-k}$$

$$Y(z)[1 + \sum_{k=1}^{N} a_k z^{-k}] = X(z) \sum_{k=0}^{M} b_k z^{-k}$$

$$\frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}} = H(z)$$

Fungsi sistem rasional

$$\frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1+\sum_{k=1}^{N} a_k z^{-k}} = H(z)$$

Pole-Zero system

Hal khusus I : $a_k = 0, \quad 1 \leq k \leq N$

$$H(z) = \sum_{k=0}^{M} b_k z^{-k} = \frac{1}{z^M} \sum_{k=0}^{M} b_k z^{M-k}$$

All-zero system

Hal khusus II : $b_k = 0, \quad 1 \leq k \leq M$

$$H(z) = \frac{b_o}{1+\sum_{k=1}^{N} a_k z^{-k}} = \frac{b_o}{\sum_{k=0}^{N} a_k z^{-k}} \quad a_o = 1$$

All-pole system

- **Contoh 1:**

Tentukan fungsi sistem dan respon impuls sistem LTI :

$$y(n) = \frac{1}{2} y(n-1) + 2x(n)$$

Jawab:

$$Y(z) = \frac{1}{2} z^{-1} Y(z) + 2X(z)$$

$$Y(z)(1 - \frac{1}{2} z^{-1}) = 2X(z)$$

$$H(z) = \frac{2}{1 - \frac{1}{2} z^{-1}} \longrightarrow \boxed{h(n) = 2\left(\frac{1}{2}\right)^{n} u(n)}$$

- **Contoh 2:**

Tentukan respon impuls dari suatu sistem LTI (Linear Time Invariant) yang dinyatakan oleh persamaan beda :

$$y(n)+3y(n-1)+y(n-2)=4{,}5x(n)+9{,}5x(n-1)$$

Jawab:

$$Y(z)+3z^{-1}Y(z)+2z^{-2}Y(z)=4{,}5X(z)+9{,}5z^{-1}X(z)$$

$$Y(z)(1+3z^{-1}+2z^{-2})=X(z)(4{,}5+9{,}5z^{-1})$$

$$H(z)=\frac{Y(z)}{X(z)}=\frac{4{,}5+9{,}5z^{-1}}{1+3z^{-1}+2z^{-2}}$$

$$H(z)=\frac{4{,}5+9{,}5z^{-1}}{1+3z^{-1}+2z^{-2}} \Longrightarrow \frac{H(z)}{z}=\frac{4{,}5z+9{,}5}{z^{2}+3z+2}$$

$$\frac{H(z)}{z}=\frac{A_1}{z+1}+\frac{A_2}{z+2}=\frac{5}{z+1}-\frac{0{,}5}{z+2}$$

$$H(z)=\frac{5}{1-(-1)z^{-1}}-\frac{0{,}5}{1-(-2)z^{-1}}$$

$$\boxed{h(n)=[5(-1)^{n}-0{,}5(-2)^{n}]u(n)}$$

- **Contoh 3:**

Tentukan output dari suatu sistem LTI (Linear Time Invariant) yang dinyatakan oleh persamaan beda :

$$y(n)+3y(n-1)+y(n-2)=4{,}5x(n)+9{,}5x(n-1)$$

$$y(-1)=0 \qquad y(-2)=0$$

dan mendapat input x(n) = $(-3)^n u(n)$ $\Longrightarrow$ $y(n)=y_{zs}(n)$

Jawab:

$$\mathrm{Y(z)} + 3\mathrm{z}^{-1}\mathrm{Y(z)} + 2\mathrm{z}^{-2}\mathrm{Y(z)} = 4{,}5\mathrm{X(z)} + 9{,}5\mathrm{z}^{-1}\mathrm{X(z)}$$

$$\mathrm{Y(z)}(1 + 3\mathrm{z}^{-1} + 2\mathrm{z}^{-2}) = \mathrm{X(z)}(4{,}5 + 9{,}5\mathrm{z}^{-1})$$

$$Y(z)(1+3z^{-1}+2z^{-2})=(4{,}5+9{,}5z^{-1})X(z)$$

$$\boxed{x(n)=(-3)^{n}u(n) \quad \rightarrow \quad X(z)=\frac{1}{1-(-3)z^{-1}}=\frac{1}{1+3z^{-1}}}$$

$$Y(z)(1+3z^{-1}+2z^{-2})=(4{,}5+9{,}5z^{-1})\frac{1}{1+3z^{-1}}$$

$$Y(z)=\frac{(4{,}5+9{,}5z^{-1})}{(1+3z^{-1}+2z^{-2})(1+3z^{-1})}$$

$$\frac{Y(z)}{z}=\frac{z^{2}(4{,}5+9{,}5z^{-1})}{z^{3}(1+3z^{-1}+2z^{-2})(1+3z^{-1})}$$

$$\frac{Y(z)}{z}=\frac{(4{,}5z^{2}+9{,}5z)}{(z^{2}+3z+2)(z+3)}=\frac{(4{,}5z^{2}+9{,}5z)}{(z+1)(z+2)(z+3)}$$

$$\frac{Y(z)}{z} = \frac{(4,5z^2 + 9,5z)}{(z^2 + 3z + 2)(z + 3)} = \frac{(4,5z^2 + 9,5z)}{(z + 1)(z + 2)(z + 3)}$$

$$\frac{(4,5z^2 + 9,5z)}{(z + 1)(z + 2)(z + 3)} = \frac{A_1}{(z + 1)} + \frac{A_2}{(z + 2)} + \frac{A_3}{(z + 3)}$$

$$\frac{Y(z)}{z} = \frac{A_1(z^2 + 5z + 6) + A_2(z^2 + 4z + 3) + A_3(z^2 + 3z + 2)}{(z + 1)(z + 2)(z + 3)}$$

$$\begin{aligned} A_1 + A_2 + A_3 &= 4,5 \\ 5A_1 + 4A_2 + 3A_3 &= 9,5 \\ 6A_1 + 3A_2 + 2A_3 &= 0 \end{aligned}$$

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 5 & 4 & 3 \\ 6 & 3 & 2 \end{vmatrix} = 2$$

$$A_1 = \frac{\begin{vmatrix} 4{,}5 & 1 & 1 \\ 9{,}5 & 4 & 3 \\ 0 & 3 & 2 \end{vmatrix}}{D} = \frac{-5}{2} = -2{,}5 \qquad A_2 = \frac{\begin{vmatrix} 1 & 4{,}5 & 1 \\ 5 & 9{,}5 & 3 \\ 6 & 0 & 2 \end{vmatrix}}{D} = \frac{2}{2} = 1$$

$$-2{,}5 + 1 + A_3 = 4{,}5 \quad \rightarrow \quad A_3 = 6$$

$$\frac{Y(z)}{z} = \frac{-2{,}5}{(z+1)} + \frac{1}{(z+2)} + \frac{6}{(z+3)}$$

$$Y(z) = \frac{-2{,}5}{(1+z^{-1})} + \frac{1}{(1+2z^{-1})} + \frac{6}{(1+3z^{-1})}$$

$$\boxed{y_{zs}(n) = [-2{,}5(-1)^n + (-2)^2 + 6(-3)^n]u(n)}$$

- **Contoh 4:**

Tentukan zero-state response dari suatu sistem LTI yang mendapat input x(n) = u(n) dan dinyatakan oleh persamaan beda :

$$y(n)+6y(n-1)+8y(n-2)=5x(n)-28x(n-1)+8x(n-2)$$

Jawab:

$$Y(z)+6z^{-1}Y(z)+8z^{-2}Y(z)=5X(z)-28z^{-1}X(z)+8z^{-2}X(z)$$

$$X(z)=\frac{1}{1-z^{-1}} \quad \Longrightarrow \quad Y(z)=\frac{(5-28z^{-1}+8z^{-2})}{1+6z^{-1}+8z^{-2}}\,\frac{1}{1-z^{-1}}$$

$$\frac{Y(z)}{z}=\frac{(5z^{2}-28z+8)}{(z^{2}+6z+8)(z-1)}=\frac{A_1}{z+2}+\frac{A_2}{z+4}+\frac{A_3}{z-1}$$

$$\frac{Y(z)}{z}=\frac{(5z^2-28z+8)}{(z+2)(z+4)(z-1)}=\frac{A_1}{z+2}+\frac{A_2}{z+4}+\frac{A_3}{z-1}$$

$$\boxed{\begin{aligned}
A_1&=\frac{(z+2)Y(z)}{z}=\left.\frac{5z^2-28z+8}{(z+4)(z-1)}\right|_{z=-2}=\frac{20+56+8}{(2)(-3)}=\frac{84}{-6}=-14\\
A_2&=\frac{(z+4)Y(z)}{z}=\left.\frac{5z^2-28z+8}{(z+2)(z-1)}\right|_{z=-4}=\frac{80+112+8}{(-2)(-5)}=\frac{200}{10}=20\\
A_3&=\frac{(z-1)Y(z)}{z}=\left.\frac{5z^2-28z+8}{(z+2)(z+4)}\right|_{z=1}=\frac{5-28+8}{(3)(5)}=\frac{-15}{15}=-1
\end{aligned}}$$

$$\frac{Y(z)}{z}=\frac{-14}{z+2}+\frac{20}{z+4}+\frac{-1}{z-1}\quad\Rightarrow\quad Y(z)=\frac{-14}{1+2z^{-1}}+\frac{20}{1+4z^{-1}}+\frac{-1}{1-z^{-1}}$$

$$y_{zs}(n)=[-14(-2)^n+20(-4)^n-1]\,u(n)$$

- **Contoh 5:**

Tentukan output dari suatu sistem LTI (Linear Time Invariant) yang dinyatakan oleh persamaan beda :

$$y(n)+3y(n-1)+y(n-2)=4{,}5x(n)+9{,}5x(n-1)$$

$$y(-1)=-8{,}5 \qquad y(-2)=7{,}5$$

dengan input x(n) = 0 → $y(n)=y_{zi}(n)$

Jawab:

$$Y^{+}(z)+3z^{-1}[Y^{+}(z)+y(-1)z]$$

$$+2z^{-2}[Y^{+}(z)+y(-1)z+y(-2)z^{2}]=0$$

$$\frac{Y^+(z)}{z} = \frac{10{,}5z+17}{z^2+3z+2} = \frac{10{,}5z+17}{(z+1)(z+2)} = \frac{A_1}{z+1} + \frac{A_2}{z+2}$$

$$\boxed{\begin{aligned} A_1 &= \frac{(z+1)Y^+(z)}{z} = \left.\frac{10{,}5z+17}{z+2}\right|_{z=-1} = \frac{6{,}5}{1} = 6{,}5 \\ A_2 &= \frac{(z+2)Y^+(z)}{z} = \left.\frac{10{,}5z+17}{z+1}\right|_{z=-2} = \frac{-4}{-1} = 4 \end{aligned}}$$

$$Y^+(z) = \frac{6{,}5z}{z+1} + \frac{4z}{z+2} = \frac{6{,}5}{1+z^{-1}} + \frac{4}{1+2z^{-1}}$$

$$\boxed{y_{zi}(n) = 6{,}5(-1)^n + 4(-2)^n}$$

- **Contoh 6:**

Tentukan output dari suatu sistem LTI yang mendapat input x(n) = u(n) dan dinyatakan oleh persamaan beda :

$$y(n)+6y(n-1)+8y(n-2)=5x(n)-28x(n-1)+8x(n-2)$$

$$y(-1)=-4 \quad y(-2)=3$$

Jawab:

$$Y^+(z)+6z^{-1}[Y^+(z)+y(-1)z]$$

$$+8z^{-2}[Y^+(z)+y(-1)z+y(-2)z^2]=5X^+(z)$$

$$-28z^{-1}[X^+(z)+x(-1)z]+8z^{-2}[X^+(z)+x(-1)z+x(-2)z^2]$$

$$Y^+(z)[1+6z^{-1}+8z^{-2}]-24-32z^{-1}+24$$

$$=X^+(z)[5-28z^{-1}+8z^{-2}]$$

$$Y^{+}(z)[1+6z^{-1}+8z^{-2}]-24-32z^{-1}+24$$

$$=X^{+}(z)[5-28z^{-1}+8z^{-2}]$$

$$Y^{+}(z)[1+6z^{-1}+8z^{-2}]=32z^{-1}+\frac{5-28z^{-1}+8z^{-2}}{1-z^{-1}}$$

$$Y^{+}(z)=\frac{32z^{-1}-32z^{-2}+5-28z^{-1}+8z^{-2}}{(1+6z^{-1}+8z^{-2})(1-z^{-1})}$$

$$Y^{+}(z)=\frac{5+4z^{-1}-24z^{-2}}{(1+6z^{-1}+8z^{-2})(1-z^{-1})}$$

$$\frac{Y^{+}(z)}{z}=\frac{5z^{2}+4z-24}{(z^{2}+6z+8)(z-1)}$$

$$\frac{Y^{+}(z)}{z} = \frac{5z^{2} + 4z - 24}{(z^{2} + 6z + 8)(z - 1)} = \frac{5z^{2} + 4z - 24}{(z + 2)(z + 4)(z - 1)}$$

$$\frac{5z^{2} - 4z - 24}{(z + 2)(z + 4)(z - 1)} = \frac{A_{1}}{z - 1} + \frac{A_{2}}{z + 2} + \frac{A_{3}}{z + 4}$$

$$A_{1} = \left.\frac{5z^{2} + 4z - 24}{(z + 2)(z + 4)}\right|_{z=1} = \frac{5 + 4 - 24}{(3)(5)} = \frac{-15}{15} = -1$$

$$A_{2} = \left.\frac{5z^{2} + 4z - 24}{(z + 4)(z - 1)}\right|_{z=-2} = \frac{20 - 8 - 24}{(2)(-3)} = \frac{-12}{-6} = 2$$

$$A_{3} = \left.\frac{5z^{2} + 4z - 24}{(z + 2)(z - 1)}\right|_{z=-4} = \frac{80 - 16 - 24}{(-2)(-5)} = \frac{40}{10} = 4$$

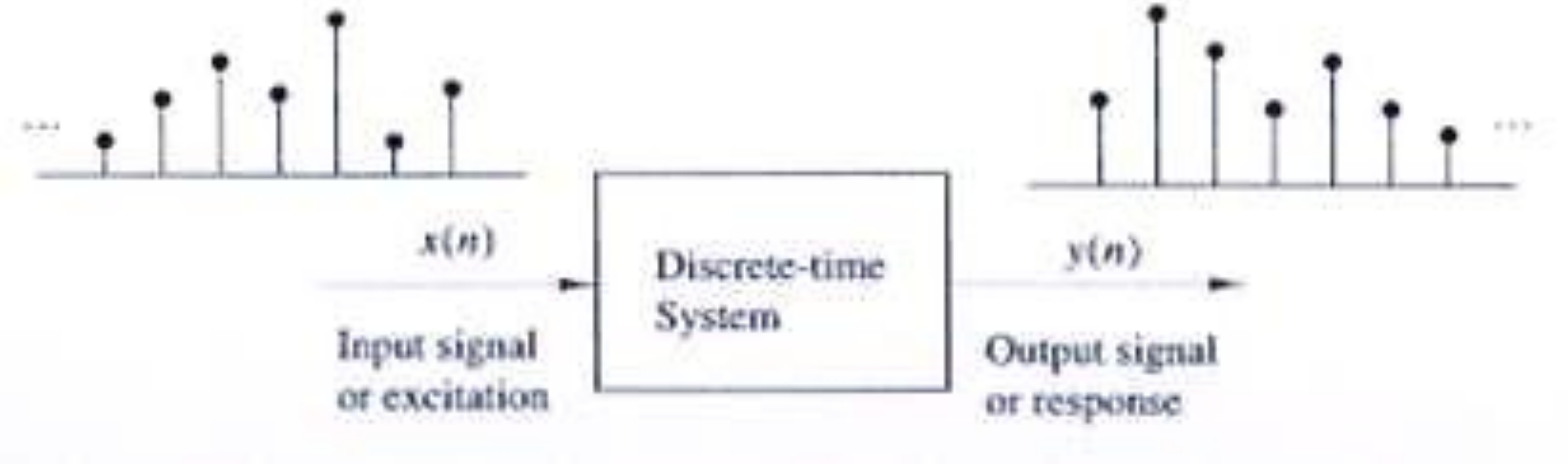
$$\frac{Y^{+}}{z} = \frac{-1}{z-1} + \frac{2}{z+2} + \frac{4}{z+4}$$

$$Y^{+} = \frac{-1}{1-z^{-1}} + \frac{2}{1+2z^{-1}} + \frac{4}{1+4z^{-1}}$$

$$y(n) = [-1 + 2(-2)^{n} + 4(-4)^{2}]u(n)$$

Deskripsi Input-Output

- Ekspresi matematik :
  - Hubungan antara input dan output
- x(n) = input (masukan, eksitasi)
- y(n) = output (keluaran, respon)
- T = Transformasi (operator)
- Sistem dipandang sebagai black box



$$y(n) = T\,[x(n)]$$

$$x(n) \quad \xrightarrow{T} \quad y(n)$$

- **Contoh 7:**

Tentukan respon dari sistem-sistem berikut terhadap input :

$$x(n) = \begin{cases} |n|, & -3 \leq n \leq 3 \\ 0, & n\ lainnya \end{cases}$$

$$a) \quad y(n) = x(n)$$

$$b) \quad y(n) = x(n-1)$$

$$c) \quad y(n) = \frac{1}{3}\left[x(n-1) + x(n) + x(n+1)\right]$$

Jawab :

$$x(n)=\left\{\cdots 0,\ 3,\ 2,\ 1,\ \underset{\uparrow}{0},\ 1,\ 2,\ 3,\ 0,\ \cdots\right\}$$

$a)\quad y(n)=x(n) \Longrightarrow$ **Sistem identitas**

$$b)\quad y(n)=x(n-1) \Longrightarrow y(0)=x(0-1)=x(-1)$$

$$y(n)=\left\{\cdots 0,\ 3,\ 2,\ \underset{\uparrow}{1},\ 0,\ 1,\ 2,\ 3,\ 0,\ \cdots\right\}$$

$$x(n)=\left\{\cdots 0,3,2,1,\underset{\uparrow}{0},1,2,3,0,\cdots\right\}$$

$$c)\quad y(n)=\frac{1}{3}\left[x(n-1)+x(n)+x(n+1)\right]$$

$$y(0)=\frac{1}{3}\left[x(-1)+x(0)+x(1)\right]$$

$$y(n)=\left\{\cdots 0,1,\frac{5}{3},2,1,\underset{\uparrow}{\frac{2}{3}},1,2,\frac{5}{3},1,0,\cdots\right\}$$

$$y(n) = \sum_{k=-\propto}^{n} x(k) \longrightarrow y(n) = \sum_{k=-\propto}^{n-1} x(k) + x(n)$$

$y(n) = y(n-1) + x(n)$ → **Akumulator**

- y(n) tidak hanya tergantung pada input x(n) tapi juga pada respon sistem sebelumnya
- y(n-1) → initial condition (kondisi awal)
- y(n-1) = 0 → sistem relaks

Contoh 8:

Tentukan respon dari akumulator dengan input
x(n) = n u(n) bila :

a) y(- 1) = 0 (sistem relaks)
b) y(- 1) = 1

Jawab :

$$y(n)=\sum_{k=-\propto}^{n}x(k)=\sum_{k=-\propto}^{-1}x(k)+\sum_{k=0}^{n}x(k)$$

$$y(n)=y(-1)+\sum_{k=0}^{n}x(k)$$

$$y(n) = y(-1) + \sum_{k=0}^{n} x(k) \qquad \sum_{k=0}^{n} x(k) = \sum_{k=0}^{n} k \; = \frac{n(n+1)}{2}$$

$$a) \quad y(-1) = 0 \quad \rightarrow \quad y(n) = \frac{n(n+1)}{2} \quad n \geq 0$$

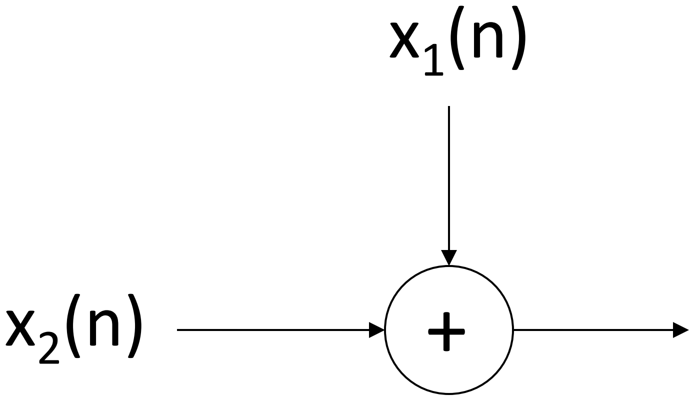
$$b) \quad y(-1) = 1 \quad \rightarrow \quad y(n) = 1 + \frac{n(n+1)}{2}$$

$$= \frac{n^2 + n + 2}{2} \quad n \geq 0$$

Representasi Diagram Blok

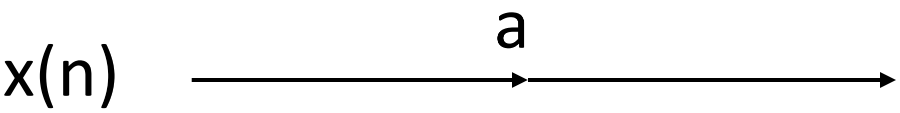
- Penjumlah (adder)
- Pengali dengan konstanta (constant muliplier)
- Pengali sinyal (signal multiplier)
- Elemen tunda (unit delay element)

Adder :



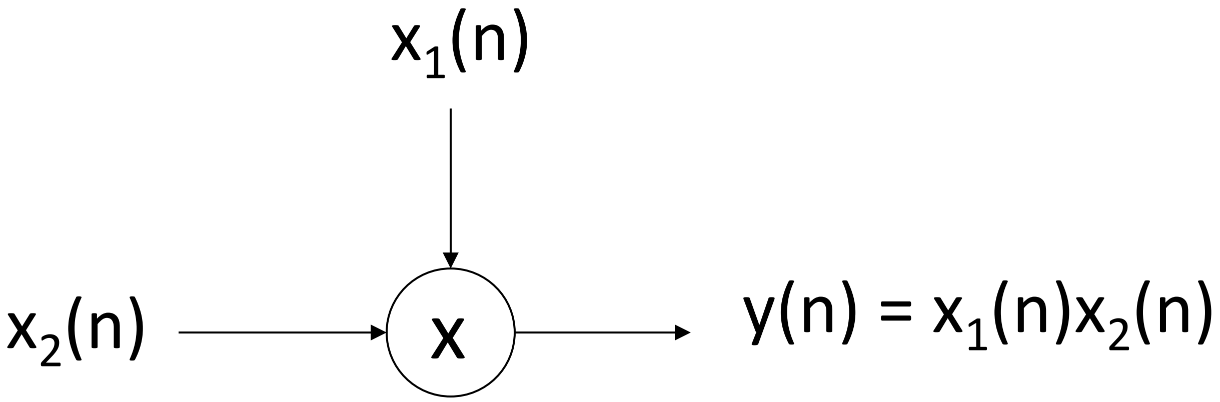
$y(n) = x_1(n) + x_2(n)$

Constant multiplier :



$y(n) = a\,x(n)$

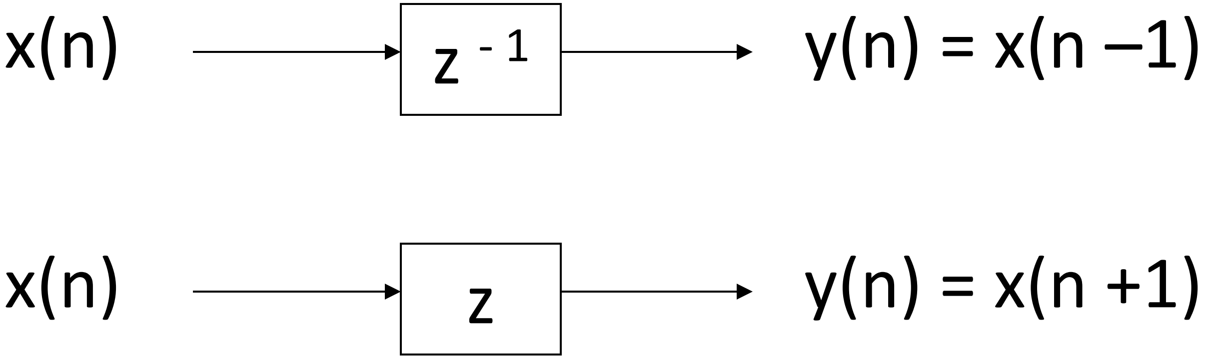
Signal multiplier :



$x_1(n)$

$x_2(n)$ → x → $y(n) = x_1(n)x_2(n)$

Unit delay element :



$x(n)$ → z^{-1} → $y(n) = x(n-1)$

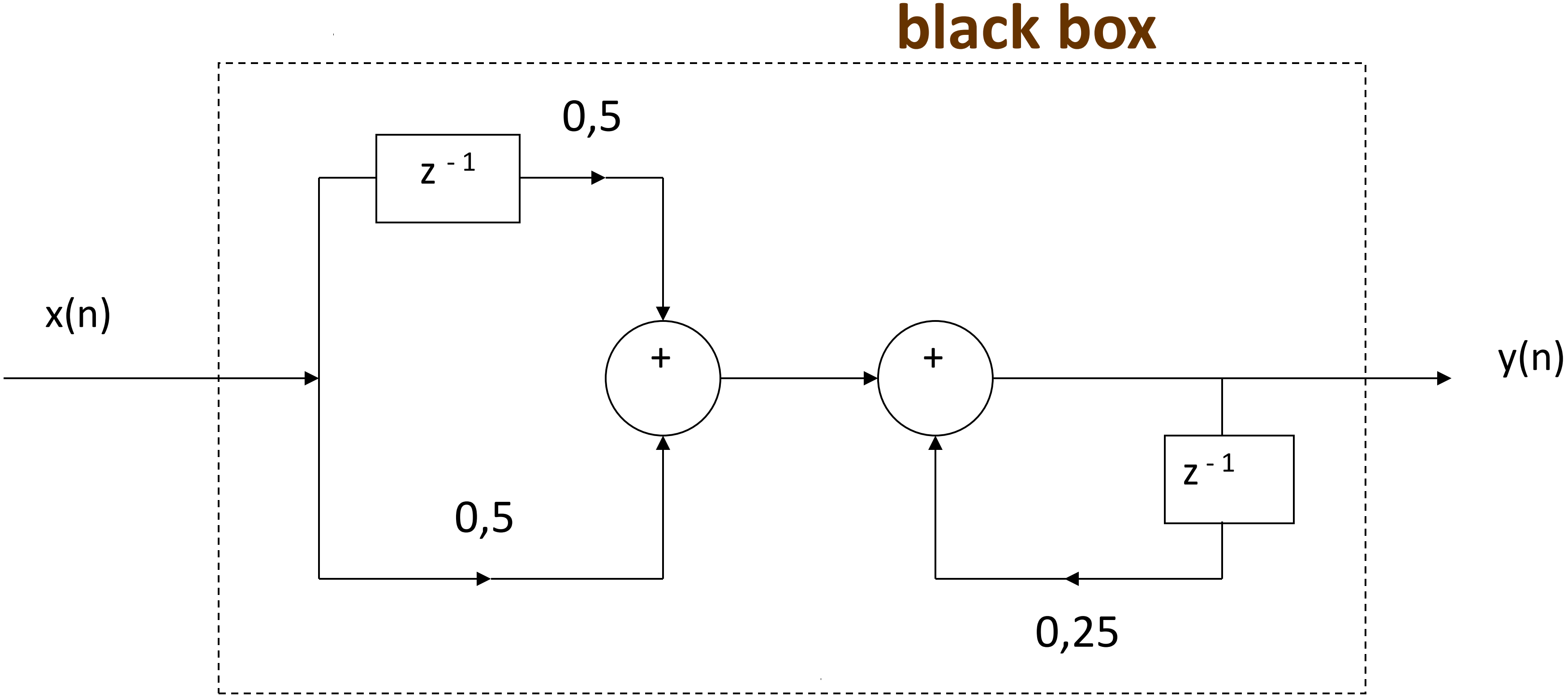
$x(n)$ → z → $y(n) = x(n+1)$

- **Contoh 9:**

Buat diagram blok dari sistem waktu diskrit dimana :

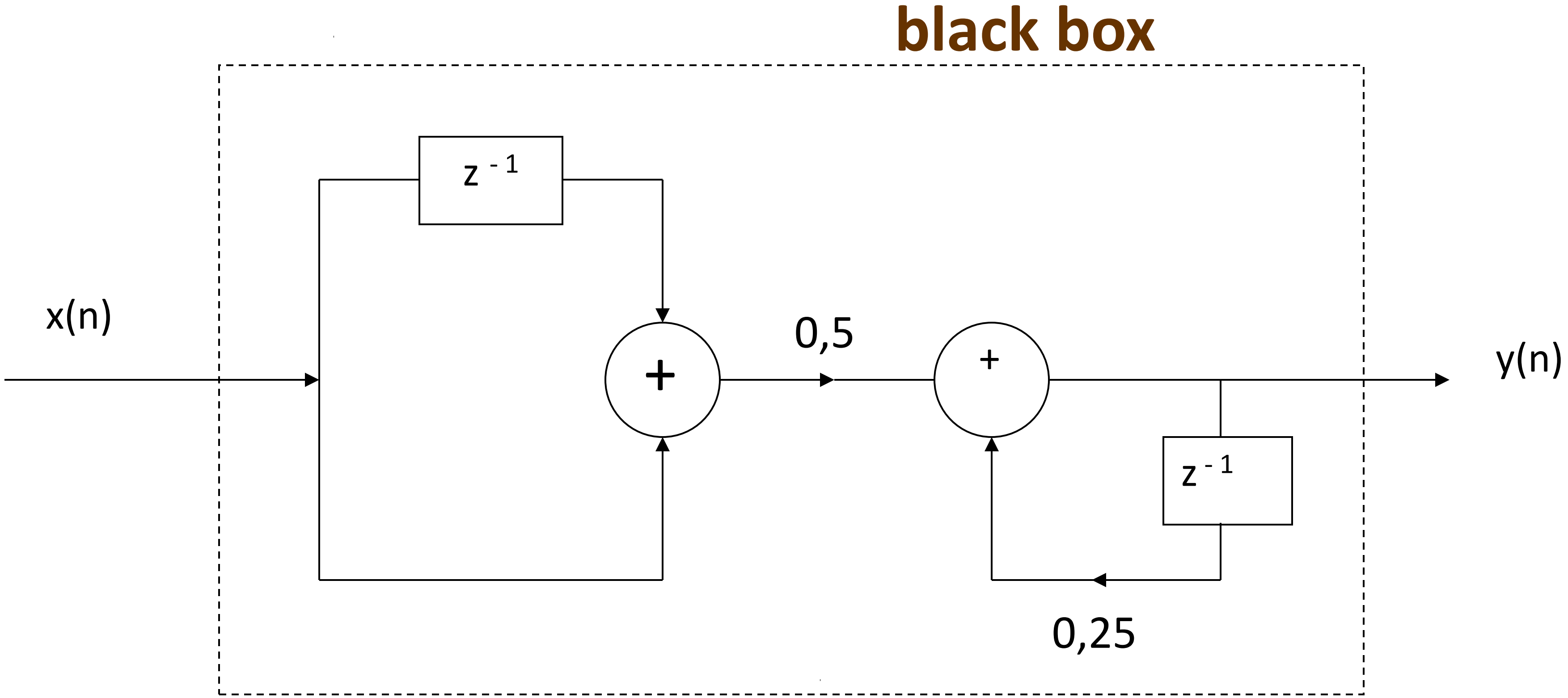
$$y(n) = \frac{1}{4}y(n-1) + \frac{1}{2}x(n) + \frac{1}{2}x(n-1)$$

Jawab :



$$y(n)=\frac{1}{4}y(n-1)+\frac{1}{2}x(n)+\frac{1}{2}x(n-1)$$

$$y(n)=\frac{1}{4}y(n-1)+\frac{1}{2}[x(n)+x(n-1)]$$



Klasifikasi Sistem

- Sistem statik dan dinamik
- Time-invariant & time-variant system
- Sistem linier dan sistem nonlinier
- Sistem kausal dan sistem nonkausal
- Sistem stabil dan sistim tak stabil

Sistem Statik (memoryless) :

- **Output pada setiap saat hanya tergantung input pada saat yang sama**
- **Tidak tergantung input pada saat yang lalu atau saat yang akan datang**

$$y(n) = a\,x(n)$$

$$y(n) = n\,x(n) + b\,x^3(n)$$

$$y(n) = T[x(n), n]$$

Sistem Dinamik :

- **Outputnya selain tergantung pada input saat yang sama juga tergantung input pada saat yang lalu atau saat yang akan datang**

$y(n) = x(n) + 3x(n-1)$ → Memori terbatas

$y(n) = \sum_{k=0}^{n} x(n-k)$ → Memori terbatas

$y(n) = \sum_{k=0}^{\propto} x(n-k)$ → Memori tak terbatas

Sistem Time-Invariant (shift-invariant) :

- **Hubungan antara input dan output tidak tergantung pada waktu**

$$y(n) = T[x(n)] \longrightarrow y(n-k) = T[x(n-k)]$$

Umumnya : $y(n,k) = T[x(n-k)]$

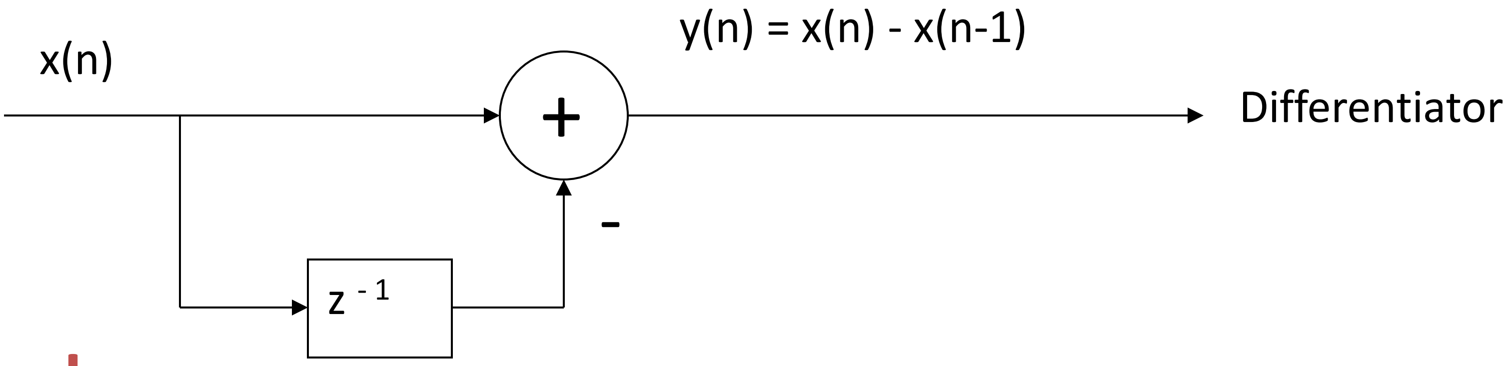
$$y(n,k) = y(n-k) \longrightarrow \text{Time-invariant}$$

$$y(n,k) \neq y(n-k) \longrightarrow \text{Time-variant}$$

- **Contoh 10:**

Tentukan apakah sistem-sistem di bawah ini time-invariant atau time-variant

a)



Jawab :

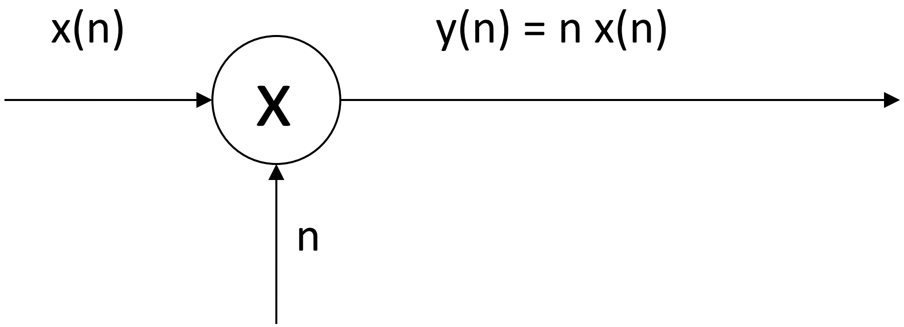
$$y(n) = T[x(n)] = x(n) - x(n-1)$$

$$y(n,k) = T[x(n-k)] = x(n-k) - x(n-k-1)$$

$$y(n-k) = x(n-k) - x(n-k-1)$$

$y(n,k) = y(n-k)$ → **Time-invariant**

b)



Time multiplier

Jawab :

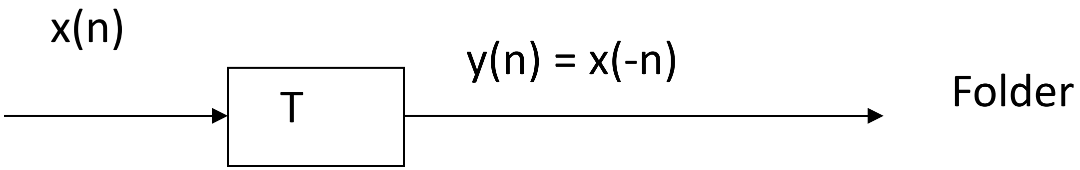
$$y(n) = T[x(n)] = nx(n)$$

$$y(n,k) = T[x(n-k)] = nx(n-k)$$

$$y(n-k) = (n-k)x(n-k) = nx(n-k) - kx(n-k)$$

$$y(n,k) \neq y(n-k) \Longrightarrow$$ **Time-variant**

c)



Jawab :

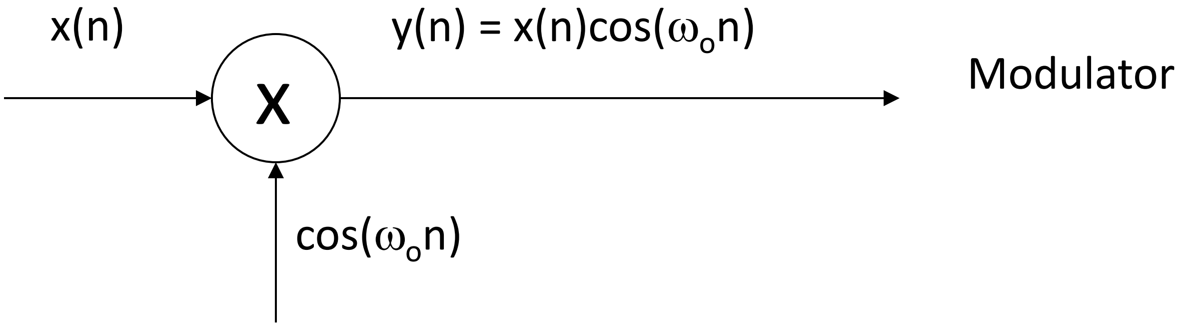
$$y(n) = T[x(n)] = x(-n)$$

$$y(n,k) = T[x(n-k)] = x(-n-k)$$

$$y(n-k) = x[-(n-k)] = x(-n+k)$$

$$y(n,k) \neq y(n-k) \Longrightarrow \textbf{Time-variant}$$

d)



x(n)

X

$y(n) = x(n)\cos(\omega_o n)$

Modulator

$\cos(\omega_o n)$

Jawab :

$$y(n) = T[x(n)] = x(n)\cos(\omega_o n)$$

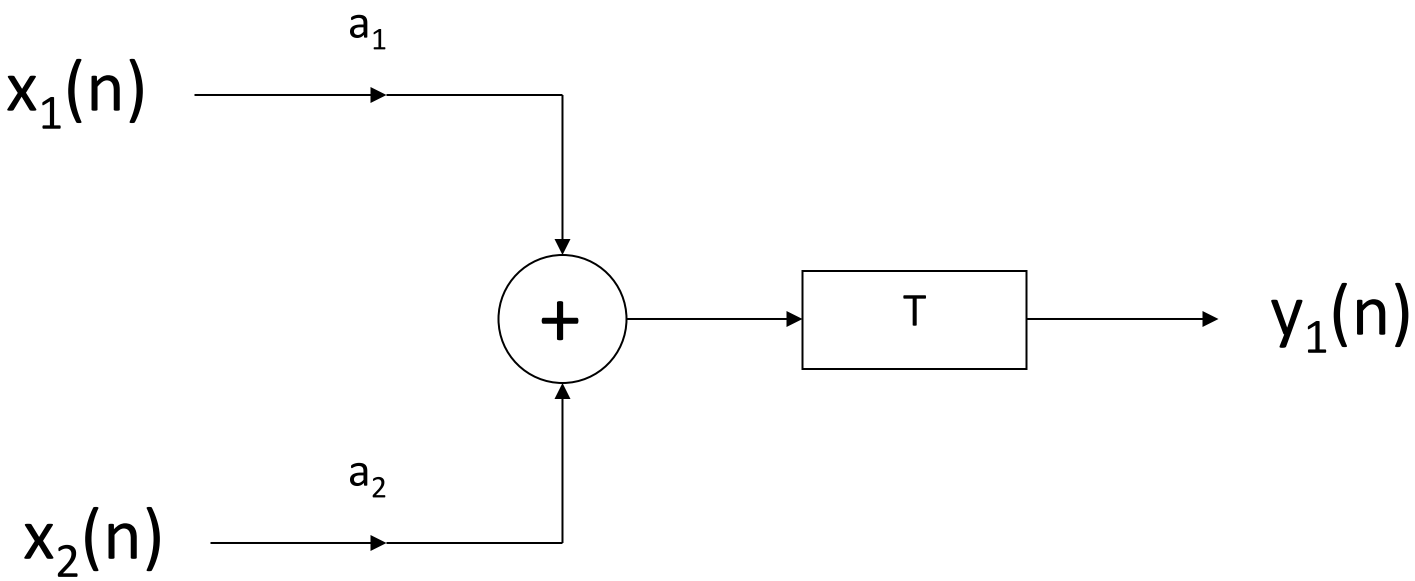
$$y(n,k) = T[x(n-k)] = x(n-k)\cos(\omega_o n)$$

$$y(n-k) = x(n-k)\cos[\omega_o (n-k)]$$

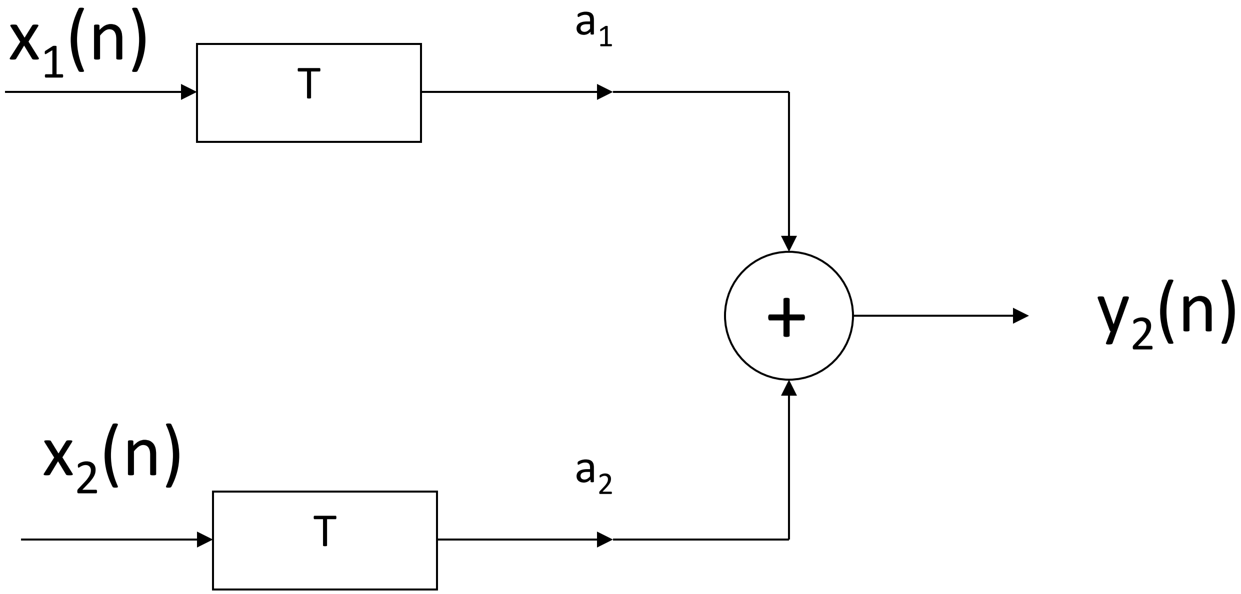
$y(n,k) \neq y(n-k)$ ⟶ **Time-variant**

Sistem Linier :

- **Prinsip superposisi berlaku**



$$y_1(n) = T[a_1 x_1(n) + a_2 x_2(n)]$$



$$y_2(n) = a_1 T[x_1(n)] + a_2 T[x_2(n)]$$

$$y_1(n) = y_2(n) \Longrightarrow$$ **Linier**

- **Contoh 11:**

Tentukan apakah sistem-sistem di bawah ini linier atau nonlinier

$$a)\quad y(n) = nx(n)$$

$$b)\quad y(n) = x(n^2)$$

$$c)\quad y(n) = x^2(n)$$

$$d)\quad y(n) = Ax(n) + B$$

Sistem Kausal :

- Outputnya hanya tergantung pada input sekarang dan input yang lalu
  - x(n), x(n-1), x(n-2), …..
- Outputnya tidak tergantung pada input yang lalu
  - x(n+1), x(n+2), …..

$$y(n) = F[x(n), x(n-1), x(n-2), \cdots]$$

- **Contoh 12:**

Tentukan kausalitas dari sistem-sistem di bawah ini :

$$a)\quad y(n) = x(n) - x(n-1)$$

$$b)\quad y(n) = \sum_{k=-\infty}^{n} x(k)$$

a, b dan c kausal

$$c)\quad y(n) = a\,x(n-k)$$

$$d)\quad y(n) = x(n) + 3x(n+4)$$

$$e)\quad y(n) = x(n^2)$$

d, e dan f nonkausal

$$f)\quad y(n) = x(2n)$$

$$g)\quad y(n) = x(-n)$$

g kausal

Sistem Stabil :

- **Setiap input yang terbatas (bounded input) akan menghasilkan output yang terbatas (bounded output) → BIBO**

$$\left|x(n)\right| \le M_x \le \propto \quad \longrightarrow \quad \left|y(n)\right| \le M_y \le \propto$$

- **Contoh 13:**

Tentukan kestabilan dari sistem di bawah ini

$$y(n) = y^2(n-1) + x(n) \qquad y(-1) = 0$$

bila mendapat input x(n) = C δ(n), 1 < C < $\propto$

Jawab :

$$y(0) = C$$

$$y(1) = C^2$$

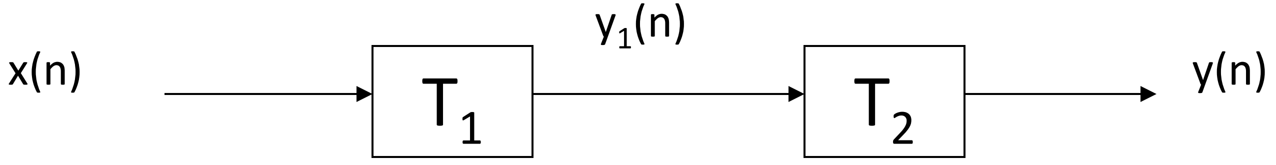
$$y(2) = C^4$$

$$y(n) = C^{2n} \quad \Longrightarrow$$

Tidak stabil

Hubungan Antar Sistem

- **Sistem-sistem kecil dapat digabungkan menjadi sistem yang lebih besar**
- **Hubungan seri dan paralel**



$$y_1(n) = T_1[x(n)]$$

$$y(n) = T_2[y_1(n)] = T_2\{T_1[x(n)]\}$$

$$T_c = T_2 T_1 \quad \rightarrow \quad y(n) = T_c[x(n)]$$

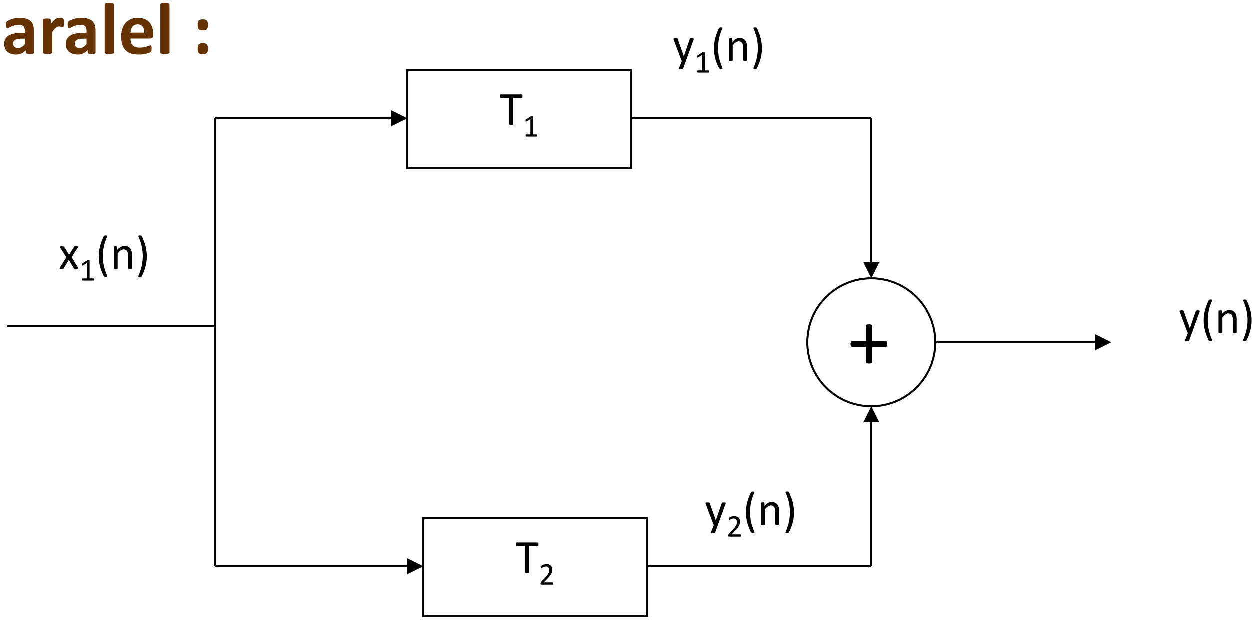
Umumnya :

$$T_2T_1 \neq T_1T_2$$

Sistem linier dan time-invariant :

$$T_2T_1 = T_1T_2$$

Hubungan paralel :



$$y(n) = y_1(n) + y_2(n) = T_1[x(n)] + T_2[x(n)]$$

$$T_p = (T_1 + T_2)$$

$$y(n) = (T_1 + T_2)[x(n)] = T_p[x(n)]$$