TIPE FILTER FASA LINIER

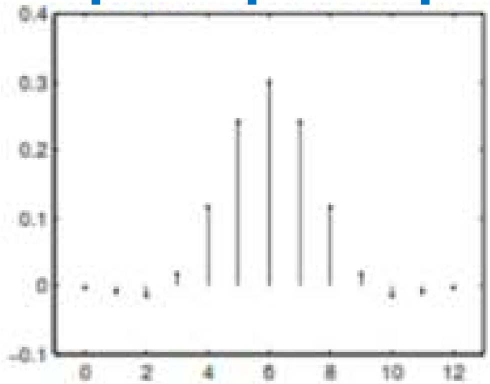
TIPE	RESPON IMPULS	
I	SIMETRI	GANJIL
II	SIMETRI	GENAP
III	ANTISIMETRI	GANJIL
IV	ANTISIMETRI	GENAP

Respon impulse untuk simetri dapat dituliskan:
$h(n) = h(N-1-n)$

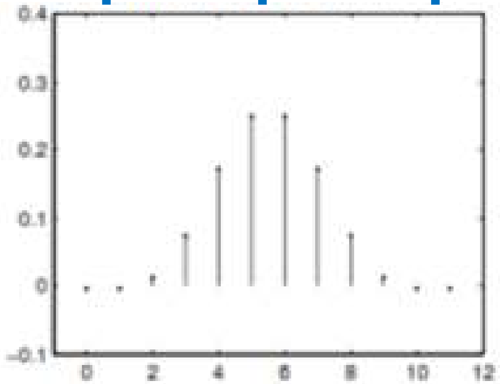
Sedangkan antisimetri:
$h(n) = -h(N-1-n)$

Jika h(n) bernilai kompleks, agar berfasa linier maka h(n) harus di conjugate-simmetry atau conjugate-anti-simmetry

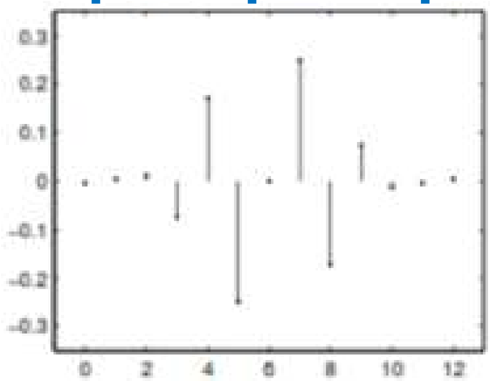
Respon impulse tipe 1



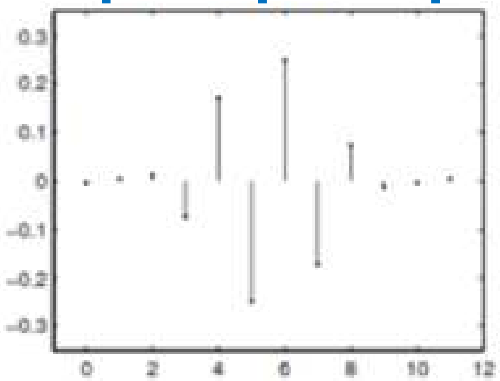
Respon impulse tipe 2



Respon impulse tipe 3



Respon impulse tipe 4



FIR fasa linier tipe 1 dengan panjang N=5:

Respon frekuensi:

$$
\begin{aligned}
H\left(e^{j\omega}\right) &= h_0 + h_1e^{-j\omega} + h_2e^{-2j\omega} + h_1e^{-3j\omega} + h_0e^{-4\omega} \\
&= e^{-2j\omega}\left(h_0e^{2j\omega} + h_1e^{j\omega} + h_2 + h_1e^{-j\omega} + h_0e^{-2j\omega}\right) \\
&= e^{-2j\omega}\left(h_0(e^{2j\omega} + e^{-2j\omega}) + h_1(e^{j\omega} + e^{-j\omega}) + h_2\right) \\
&= e^{-2j\omega}\left(2h_0\cos\left(2\omega\right) + 2h_1\cos\left(\omega\right) + h_2\right) \\
&= A(\omega)e^{j\theta(\omega)}
\end{aligned}
$$

dimana:

$$\theta(\omega) = -2\omega$$

$$A(\omega) = 2h_0\cos\left(2\omega\right) + 2h_1\cos\left(\omega\right) + h_2$$ Nilai A ini real

FIR fasa linier tipe 1 secara umum dengan panjang N:

Respon frekuensi:

$$H\left(e^{j\omega}\right) = A(\omega)e^{j\theta(\omega)}$$

Respon magnitude:

$$A(\omega) = h(M) + 2\sum_{n=0}^{M-1} h(n)\cos\left((M-n)\omega\right)$$

Respon fasa:

$$\theta(\omega) = -M\omega$$

$$M = \frac{N-1}{2}$$

FIR fasa linier tipe 2 dengan panjang N=4:

Respon frekuensi:

$$\begin{aligned}
H(e^{j\omega}) &= h_0 + h_1 e^{-j\omega} + h_1 e^{-2j\omega} + h_0 e^{-3j\omega} \\
&= e^{-\frac{3}{2}j\omega}\left(h_0 e^{\frac{3}{2}j\omega} + h_1 e^{\frac{1}{2}j\omega} + h_1 e^{-\frac{1}{2}j\omega} + h_0 e^{-\frac{3}{2}j\omega}\right) \\
&= e^{-\frac{3}{2}j\omega}\left(h_0(e^{\frac{3}{2}j\omega} + e^{-\frac{3}{2}j\omega}) + h_1(e^{\frac{1}{2}j\omega} + e^{-\frac{1}{2}j\omega})\right) \\
&= e^{-\frac{3}{2}j\omega}\left(2h_0 \cos\left(\tfrac{3}{2}\omega\right) + 2h_1 \cos\left(\tfrac{1}{2}\omega\right)\right) \\
&= A(\omega)e^{j\theta(\omega)}
\end{aligned}$$

dimana:

$$\theta(\omega) = -\frac{3}{2}\omega$$

$$A(\omega) = 2h_0 \cos\left(\tfrac{3}{2}\omega\right) + 2h_1 \cos\left(\tfrac{1}{2}\omega\right)$$

Nilai A ini real

FIR fasa linier tipe 2 secara umum dengan panjang N:

Respon frekuensi:

$$H(e^{j\omega}) = A(\omega)e^{j\theta(\omega)}$$

Respon magnitude:

$$A(\omega) = 2\sum_{n=0}^{\frac{N}{2}-1} h(n)\cos((M-n)\omega)$$

Respon fasa:

$$\theta(\omega) = -M\omega$$

$$M = \frac{N-1}{2}$$

FIR fasa linier tipe 3 dengan panjang N=4:

Respon frekuensi:

$$\begin{aligned}
H\left(e^{j\omega}\right) &= h_0 + h_1 e^{-j\omega} - h_1 e^{-3j\omega} - h_0 e^{-4\omega} \\
&= e^{-2j\omega}\left(h_0 e^{2j\omega} + h_1 e^{j\omega} - h_1 e^{-j\omega} - h_0 e^{-2j\omega}\right) \\
&= e^{-2j\omega}\left(h_0(e^{2j\omega} - e^{-2j\omega}) + h_1(e^{j\omega} - e^{-j\omega})\right) \\
&= e^{-2j\omega}\left(2jh_0 \sin(2\omega) + 2jh_1 \sin(\omega)\right) \\
&= e^{-2j\omega} j \left(2h_0 \sin(2\omega) + 2h_1 \sin(\omega)\right) \\
&= e^{-2j\omega} e^{j\frac{\pi}{2}} \left(2h_0 \sin(2\omega) + 2h_1 \sin(\omega)\right) \\
&= A(\omega) e^{j\theta(\omega)}
\end{aligned}$$

dimana:

$$\theta(\omega) = -2\omega + \frac{\pi}{2}$$

$$A(\omega) = 2h_0 \sin(2\omega) + 2h_1 \sin(\omega)$$

Nilai A ini real

FIR fasa linier tipe 3 secara umum dengan panjang N:

Respon frekuensi: $H(e^{j\omega}) = A(\omega)e^{j\theta(\omega)}$

Respon magnitude: $A(\omega) = 2\sum_{n=0}^{M-1} h(n)\sin((M-n)\omega)$

Respon fasa: $\theta(\omega) = -M\omega + \frac{\pi}{2}$

$$M = \frac{N-1}{2}$$

FIR fasa linier tipe 4 dengan panjang N=5:

Respon frekuensi:

$$
\begin{aligned}
H\left(e^{j\omega}\right) &= h_0 + h_1 e^{-j\omega} - h_1 e^{-2j\omega} - h_0 e^{-3j\omega} \\
&= e^{-\frac{3}{2}j\omega}\left(h_0 e^{\frac{3}{2}j\omega} + h_1 e^{\frac{1}{2}j\omega} - h_1 e^{-\frac{1}{2}j\omega} - h_0 e^{-\frac{3}{2}j\omega}\right) \\
&= e^{-\frac{3}{2}j\omega}\left(h_0(e^{\frac{3}{2}j\omega} - e^{-\frac{3}{2}j\omega}) + h_1(e^{\frac{1}{2}j\omega} - e^{-\frac{1}{2}j\omega})\right) \\
&= e^{-\frac{3}{2}j\omega}\left(2jh_0 \sin\left(\tfrac{3}{2}\omega\right) + 2jh_1 \sin\left(\tfrac{1}{2}\omega\right)\right) \\
&= e^{-\frac{3}{2}j\omega} j \left(2h_0 \sin\left(\tfrac{3}{2}\omega\right) + 2h_1 \sin\left(\tfrac{1}{2}\omega\right)\right) \\
&= e^{-\frac{3}{2}j\omega} e^{j\frac{\pi}{2}} \left(2h_0 \sin\left(\tfrac{3}{2}\omega\right) + 2h_1 \sin\left(\tfrac{1}{2}\omega\right)\right) \\
&= A(\omega)e^{j\theta(\omega)}
\end{aligned}
$$

dimana:

$$\theta(\omega) = -\frac{3}{2}\omega + \frac{\pi}{2}$$

$$A(\omega) = 2h_0 \sin\left(\tfrac{3}{2}\omega\right) + 2h_1 \sin\left(\tfrac{1}{2}\omega\right)$$ Nilai A ini real

FIR fasa linier tipe 4 secara umum dengan panjang N:

Respon frekuensi:

$$H\left(e^{j\omega}\right) = A(\omega)\, e^{j\theta(\omega)}$$

Respon magnitude:

$$A(\omega) = 2\sum_{n=0}^{\frac{N}{2}-1} h(n)\sin\left((M-n)\omega\right)$$

Respon fasa:

$$\theta(\omega) = -M\omega + \frac{\pi}{2}$$

$$M = \frac{N-1}{2}$$

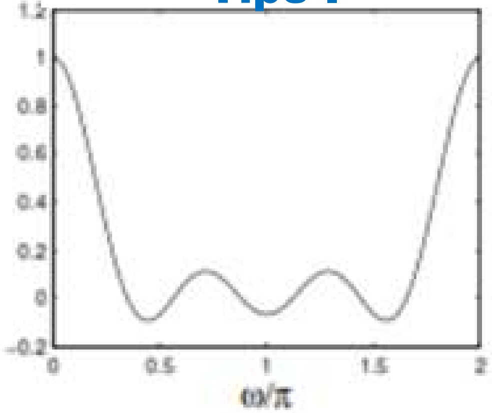
RINGKASAN RESPON MAGNITUDA, FASA UNTUK FIR FASA LINIER

Type	$\theta(\omega)$	$A(\omega)$
I	$-M\omega$	$h(M) + 2\sum_{n=0}^{M-1} h(n)\cos((M-n)\omega)$
II	$-M\omega$	$2\sum_{n=0}^{\frac{N}{2}-1} h(n)\cos((M-n)\omega)$
III	$-M\omega + \frac{\pi}{2}$	$2\sum_{n=0}^{M-1} h(n)\sin((M-n)\omega)$
IV	$-M\omega + \frac{\pi}{2}$	$2\sum_{n=0}^{\frac{N}{2}-1} h(n)\sin((M-n)\omega)$

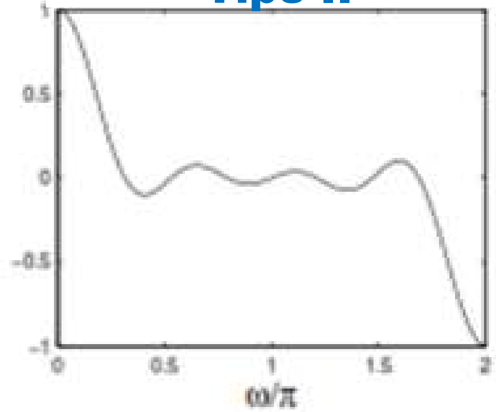
$$M = \frac{N-1}{2}$$

KARAKTERISTIK RESPON AMPLITUDA

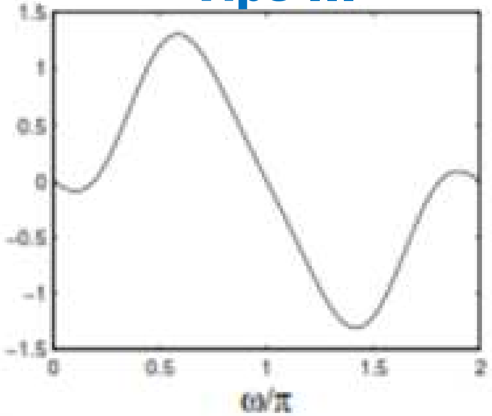
Respon Amplituda Tipe I



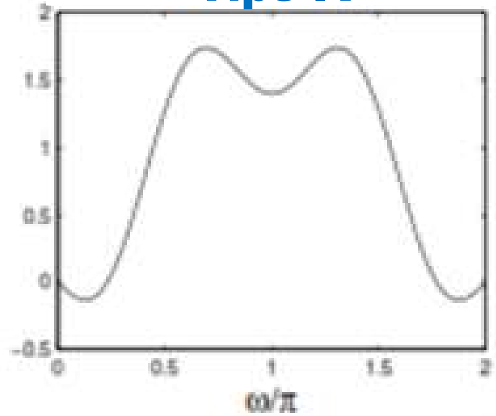
Respon Amplituda Tipe II



Respon Amplituda Tipe III



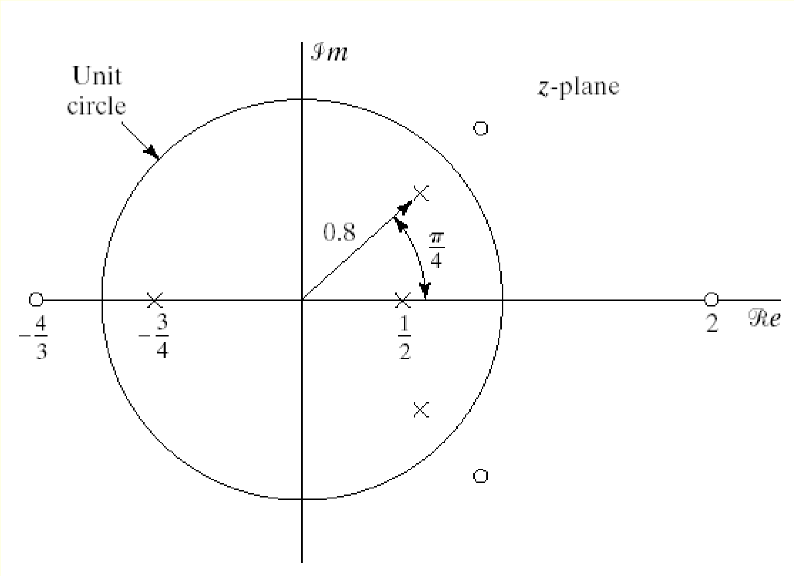
Respon Amplituda Tipe IV



All-Pass System

- A system with frequency response magnitude constant
- Important uses such as compensating for phase distortion
- Simple all-pass system

$$H_{ap}(z) = \frac{z^{-1} - a^*}{1 - az^{-1}}$$



- Magnitude response constant

$$H_{ap}\left(e^{j\omega}\right) = \frac{e^{-j\omega} - a^*}{1 - ae^{-j\omega}} = e^{-j\omega}\frac{1 - a^* e^{j\omega}}{1 - ae^{-j\omega}}$$

- Most general form with real impulse response

$$H_{ap}(z) = A\prod_{k=1}^{M_r}\frac{z^{-1} - d_k}{1 - d_k z^{-1}}\prod_{k=1}^{M_c}\frac{\left(z^{-1} - c_k^*\right)\left(z^{-1} - c_k\right)}{\left(1 - c_k z^{-1}\right)\left(1 - c_k^* z^{-1}\right)}$$

- A: positive constant, d_k: real poles, c_k: complex poles

Phase of All-Pass Systems

$$H_{ap}\left(e^{j\omega}\right) = \frac{e^{-j\omega} - a^*}{1 - ae^{-j\omega}} = e^{-j\omega}\frac{1 - a^* e^{j\omega}}{1 - ae^{-j\omega}}$$

- Let's write the phase with a represented in polar form

$$\angle\left[\frac{e^{-j\omega} - re^{-j\theta}}{1 - re^{j\theta}e^{-j\omega}}\right] = -\omega - 2\arctan\left[\frac{r\sin(\omega - \theta)}{1 - r\cos(\omega - \theta)}\right]$$

- The group delay of this system can be written as

$$\text{grd}\left[\frac{e^{-j\omega} - re^{-j\theta}}{1 - re^{j\theta}e^{-j\omega}}\right] = \frac{1 - r^2}{1 - 2r\cos(\omega - \theta) + r^2} = \frac{1 - r^2}{\left|1 - re^{j\theta}e^{-j\omega}\right|^2}$$

- For stable and causal system $|r|<1$
  - Group delay of all-pass systems is always positive
- Phase between 0 and π is always negative

$$\arg\left[H_{ap}\left(e^{j\omega}\right)\right] \le 0 \quad \text{for} \quad 0 \le \omega < \pi$$

Minimum-Phase System

- A system with all poles and zeros inside the unit circle
- Both the system function and the inverse is causal and stable
- Name "minimum-phase" comes from the property of the phase
  - Not obvious to see with the given definition
  - Will look into it
- Given a magnitude square system function that is minimum phase
  - The original system is uniquely determined
- Minimum-phase and All-pass decomposition
  - Any rational system function can be decomposed as

$$H(z) = H_{min}(z)H_{ap}(z)$$

Example 1: Minimum-Phase System

- Consider the following system

$$H_1(z) = \frac{1 + 3z^{-1}}{1 + \frac{1}{2}z^{-1}}$$

- One pole inside the unit circle:
  - Make part of minimum-phase system
- One zero outside the unit circle:
  - Add an all-pass system to reflect this zero inside the unit circle

$$H_1(z) = \frac{1 + 3z^{-1}}{1 + \frac{1}{2}z^{-1}} = 3\frac{1}{1 + \frac{1}{2}z^{-1}}\left(z^{-1} + \frac{1}{3}\right) = 3\frac{1}{1 + \frac{1}{2}z^{-1}}\left(z^{-1} + \frac{1}{3}\right)\frac{1 - \frac{1}{3}z^{-1}}{1 - \frac{1}{3}z^{-1}}$$

$$H_1(z) = \left(3\frac{1 - \frac{1}{3}z^{-1}}{1 + \frac{1}{2}z^{-1}}\right)\left(\frac{z^{-1} + \frac{1}{3}}{1 - \frac{1}{3}z^{-1}}\right) = H_{min}(z)H_{ap}(z)$$

Example 2: Minimum-Phase System

- Consider the following system

$$H_2(z) = \frac{\left(1 + \frac{3}{2} e^{j\pi/4} z^{-1}\right)\left(1 + \frac{3}{2} e^{-j\pi/4} z^{-1}\right)}{1 - \frac{1}{3} z^{-1}}$$

- One pole inside the unit circle:
- Complex conjugate zero pair outside the unit circle

$$H_2(z) = \frac{\left(1 + \frac{3}{2} e^{j\pi/4} z^{-1}\right)\left(1 + \frac{3}{2} e^{-j\pi/4} z^{-1}\right)}{1 - \frac{1}{3} z^{-1}}$$

$$= \frac{\frac{3}{2} e^{j\pi/4} \frac{3}{2} e^{-j\pi/4} \left(\frac{2}{3} e^{-j\pi/4} + z^{-1}\right)\left(\frac{2}{3} e^{j\pi/4} + z^{-1}\right)}{1 - \frac{1}{3} z^{-1}}$$

Example 2 Cont'd

$$H_2(z) = \frac{\frac{9}{4}\left(\frac{2}{3}e^{-j\pi/4} + z^{-1}\right)\left(\frac{2}{3}e^{j\pi/4} + z^{-1}\right)}{1 - \frac{1}{3}z^{-1}} \cdot \frac{\left(1 + \frac{2}{3}e^{-j\pi/4}z^{-1}\right)\left(1 + \frac{2}{3}e^{j\pi/4}z^{-1}\right)}{\left(1 + \frac{2}{3}e^{-j\pi/4}z^{-1}\right)\left(1 + \frac{2}{3}e^{j\pi/4}z^{-1}\right)}$$

$$H_2(z) = \frac{\frac{9}{4}\left(1 + \frac{2}{3}e^{-j\pi/4}z^{-1}\right)\left(1 + \frac{2}{3}e^{j\pi/4}z^{-1}\right)}{1 - \frac{1}{3}z^{-1}} \cdot \frac{\left(\frac{2}{3}e^{-j\pi/4} + z^{-1}\right)\left(\frac{2}{3}e^{j\pi/4} + z^{-1}\right)}{\left(1 + \frac{2}{3}e^{-j\pi/4}z^{-1}\right)\left(1 + \frac{2}{3}e^{j\pi/4}z^{-1}\right)}$$

$$H_2(z) = H_{min}(z)H_{ap}(z)$$