Contoh perancangan IIR dengan tipe Butterworth

Rancanglah filter digital LPF IIR spesifikasinya sebagai berikut: frekuensi cut off -3 dB pada frekuensi 500 Hz, monotonik dan daerah redaman pada -15 dB pada frekuensi 750 Hz dimana frekuensi samplingnya 2000 sample/detik.

Jawab:

Kita hitung spek. filter digital :

$$\omega_P = \frac{2\pi 500}{2000} = 0{,}5\pi \text{ rad/sample}$$

$$\omega_S = \frac{2\pi 750}{2000} = 0{,}75\pi \text{ rad/sample}$$

Kita hitung spek. Filter analog :

$$\Omega_P = \frac{2}{T}\tan\left(\frac{\omega p}{2}\right) = 2.2000\tan\left(\frac{0{,}5\pi}{2}\right) = 4000 \text{ rad/detik}$$

$$\Omega_S = \frac{2}{T}\tan\left(\frac{\omega_S}{2}\right) = 2.2000\tan\left(\frac{0{,}75\pi}{2}\right) = 9656{,}85 \text{ rad/detik}$$

Kita hitung frekuensi cut off LPF ternormalisasi :

$$\Omega_c = \frac{9656{,}85}{4000} = 2{,}41$$

Kita hitung orde filter :

$$n = \left\lceil \frac{\log\left[\dfrac{10^{\frac{-Rp}{10}} - 1}{10^{\frac{-Rs}{10}} - 1}\right]}{2\log\left(\dfrac{1}{\Omega_C}\right)} \right\rceil$$

$$n = \left\lceil \frac{\log\left[\frac{10^{\frac{3}{10}}-1}{10^{\frac{15}{10}}-1}\right]}{2\log\left(\frac{1}{2,41}\right)} \right\rceil = \lceil 1,94 \rceil = 2$$

Lihat tabel butterworth untuk N=2, maka didapat fungsi alih butterworth order-2 ternormalisasi:

$$H_n(s) = \frac{1}{S^2 + 1,4142S + 1}$$

Fungsi alih butterworth order-2 untuk Ωp=4000, diperoleh menggunakan transformasi analog to analog, hasilnya:

$$H(s)|_{s\to\frac{s}{4000}} = \frac{1}{(\frac{s}{4000})^2+\sqrt{2}\frac{s}{4000}+1} = \frac{(4000)^2}{S+4000.s.\sqrt{2}+(4000)^2};$$

Tranformasi Bilinear:

$$H(z) = H(s)|_{8\to\left[\frac{2(1-z)^{-1}}{T(1+z)^{-1}}\right]} = \frac{(400)^2}{[(400)\frac{(1-z^{-1}}{(1+z^{-1})}]^2+4000\sqrt{2}(4000)\frac{(1-z^{-1})}{(1+z^{-1}}+(4000)^2}$$

$$H(z) = \frac{\left(1+z^{-1}\right)^2}{\left(1-z^{-1}\right)^2+\sqrt{2}(1+z^{-1})(1-z^{-1})+\left(1-z^{-1}\right)^2}$$

$$H(z) = \frac{1+2.z^{-1}+z^{-2}}{3.414+0.5857.z^{-2}}$$

$$H(z) = \frac{1+2.z^{-1}+z^{-2}}{3.414+0.5857.z^{-2}} \rightarrow \frac{Y(z)}{X(z)}$$

$$Y(z)\{3.414+0.5867z^{-2}\} = X(z)\{1+2z^{-1}+z^{-2}\}$$
$$3.414\,y(n)+0.5857\,y(n-2) = x(n)+2x(n-1)+x(n-2)$$
$$y(n) = \frac{1}{3.414}\left[x(n)+x(n-1)+2.x(n-2)-0.5887\,y(n-2)\right]$$

Transformasi Impulse Invariance

- Diketahui suatu fungsi alih filter analog LPF adalah sebagai beriku :

$$H(s) = \frac{1}{(s^2 + 5s + 6)}$$

Tentukan tranfers function digital H(z) dengan metode Impulse invariant dengan sampling time T = 0,1 detik

Jawab :

Uraikan H(s) dengan parsiil

$$H(s) = \frac{s+1}{s^2+5s+6} = \frac{s+1}{(s+2)(s+3)} = \frac{A}{(s+2)} + \frac{B}{(s+3)}$$

$$A(s+3) + B(s+2) = s+1$$
$$(A+B)s + (3A+2B) = s+1$$
$$A + B = 1 \quad \rightarrow A = -1, B = 2$$
$$3A + 2B = 1$$

Maka

$$H(s) = \frac{1}{(s+2)} + \frac{2}{(s+3)}$$

$$H(z) = \frac{-1}{(1-e^{-2\tau}z^{-1})} + \frac{2}{(1-e^{-3\tau}z^{-1})}$$

$$= \frac{-(1-e^{-3\tau}z^{-1})+2(1-e^{-2}z^{-1})}{(1-e^{-3\tau}z^{-1})(1-e^{-3\tau}z^{-1})}$$

$$= \frac{1+z^{-1}(e^{-3\tau}-2e^{-2\tau})}{(1-e^{-2\tau}z^{-1})(1-e^{-3\tau}z^{-1})}$$

$$= \frac{1+ \quad (0.7408-1.6374)}{(1-0{,}8187z^{-1})(1-0.7408z^{1})}$$

$$= \frac{1-0.8966z^{-1}}{(1-0{,}8187z^{-1})(1-0.7408z^{-1})} = \frac{Y(z)}{X(z)}$$

Sehingga

$$Y(z)(1-0.8187z^{-1})(10.7408z^{-1})=X(z)(1-0.8966z^{-1})$$
$$Y(z)(1-1.5595z^{-1}+0.6064z^{-2}= X(z)-0.8966z^{-1}X(z)$$
$$Y(z)-1.5595z^{-1}Y(z)+0.6064z^{-2}$$
$$= X(z)-0.8966z^{-1}X(z)$$

Persamaan bedanya:

$$y(n)=x(n)-0.8966\,x(n-1)+1.5595\,y(n-1)$$
$$-0.6064\,y\,(n-2)$$