

Contoh perancangan IIR dengan tipe Butterworth

Rancanglah filter digital LPF IIR spesifikasinya sebagai berikut: frekuensi cut off -3 dB pada frekuensi 500 Hz, monotonik dan daerah redaman pada -15 dB pada frekuensi 750 Hz dimana frekuensi samplangnya 2000 sample/detik.

Jawab:

Kita hitung spek. filter digital :

$$\omega_p = \frac{2\pi 500}{2000} = 0,5\pi \text{ rad/sample}$$

$$\omega_s = \frac{2\pi 750}{2000} = 0,75\pi \text{ rad/sample}$$

Kita hitung spek. Filter analog :

$$\Omega_p = \frac{2}{T} \tan\left(\frac{\omega_p}{2}\right) = 2.2000 \tan\left(\frac{0,5\pi}{2}\right) = 4000 \text{ rad/detik}$$

$$\Omega_s = \frac{2}{T} \tan\left(\frac{\omega_s}{2}\right) = 2.2000 \tan\left(\frac{0,75\pi}{2}\right) = 9656,85 \text{ rad/detik}$$

Kita hitung frekuensi cut off LPF ternormalisasi :

$$\Omega_c = \frac{9656,85}{4000} = 2,41$$

Kita hitung orde filter :

$$n = \frac{\log \left[\frac{10^{\frac{-R_p}{10}} - 1}{10^{\frac{-R_s}{10}} - 1} \right]}{2 \log \left(\frac{1}{\Omega_c} \right)}$$

$$n = \left\lceil \frac{\log \left[\frac{10^{\frac{3}{10}} - 1}{10^{\frac{15}{10}} - 1} \right]}{2 \log \left(\frac{1}{2,41} \right)} \right\rceil = \lceil 1,94 \rceil = 2$$

Lihat tabel butterworth untuk N=2, maka didapat fungsi alih butterworth order-2 ternormalisasi:

$$H_n(s) = \frac{1}{S^2 + 1,4142S + 1}$$

Fungsi alih butterworth order-2 untuk $\Omega_p=4000$, diperoleh menggunakan transformasi analog to analog, hasilnya:

$$H(s)|_{s \rightarrow \frac{s}{4000}} = \frac{1}{\left(\frac{s}{4000}\right)^2 + \sqrt{2}\frac{s}{4000} + 1} = \frac{(4000)^2}{s + 4000.s.\sqrt{2} + (4000)^2}$$

Tranformasi Bilinear:

$$H(z) = H(s)|_{s \rightarrow \left[\frac{2(1-z)^{-1}}{T(1+z)^{-1}} \right]} = \frac{(400)^2}{\left[(400) \frac{(1-z^{-1})}{(1+z^{-1})} \right]^2 + 4000\sqrt{2}(4000) \frac{(1-z^{-1})}{(1+z^{-1})} + (4000)^2}$$

$$H(z) = \frac{(1 + z^{-1})^2}{(1 - z^{-1})^2 + \sqrt{2}(1 + z^{-1})(1 - z^{-1}) + (1 - z^{-1})^2}$$

$$H(z) = \frac{1 + 2 \cdot z^{-1} + z^{-2}}{3.414 + 0.5857 \cdot z^{-2}}$$

$$H(z) = \frac{1 + 2 \cdot z^{-1} + z^{-2}}{3.414 + 0.5857 \cdot z^{-2}} \rightarrow \frac{Y(z)}{X(z)}$$

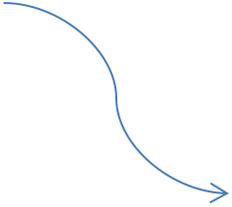
$$Y(z)\{3.414 + 0.5867z^{-2}\} = X(z)\{1 + 2z^{-1} + z^{-2}\}$$

$$3.414 y(n) + 0.5857 y(n - 2) = x(n) + 2x(n - 1) + x(n - 2)$$

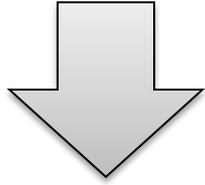
$$y(n) = \frac{1}{3.414} [x(n) + x(n - 1) + 2 \cdot x(n - 2) - 0.5887 y(n - 2)]$$

Transformasi Impulse Invariance

- Diketahui suatu fungsi alih filter analog LPF adalah sebagai berikut :


$$H(s) = \frac{1}{(s^2 + 5s + 6)}$$

Tentukan transfer function digital $H(z)$ dengan metode Impulse invariant dengan sampling time $T = 0,1$ detik



Jawab :

Uraikan $H(s)$ dengan parsial

$$H(s) = \frac{s+1}{s^2+5s+6} = \frac{s+1}{(s+2)(s+3)} = \frac{A}{(s+2)} + \frac{B}{(s+3)}$$

$$A(s+3) + B(s+2) = s+1$$

$$(A+B)s + (3A+2B) = s+1$$

$$A+B=1 \quad \rightarrow A=-1, B=2$$

$$3A+2B=1$$

Maka

$$H(s) = \frac{1}{(s+2)} + \frac{2}{(s+3)}$$

$$H(z) = \frac{-1}{(1-e^{-2\tau}z^{-1})} + \frac{2}{(1-e^{-3\tau}z^{-1})}$$

$$= \frac{-(1-e^{-3\tau}z^{-1}) + 2(1-e^{-2\tau}z^{-1})}{(1-e^{-3\tau}z^{-1})(1-e^{-2\tau}z^{-1})}$$

$$= \frac{1 + z^{-1}(e^{-3\tau} - 2e^{-2\tau})}{(1-e^{-2\tau}z^{-1})(1-e^{-3\tau}z^{-1})}$$

$$= \frac{1 + (0.7408 - 1.6374)z^{-1}}{(1 - 0,8187z^{-1})(1 - 0.7408z^{-1})}$$

$$= \frac{1 - 0.8966z^{-1}}{(1 - 0,8187z^{-1})(1 - 0.7408z^{-1})} = \frac{Y(z)}{X(z)}$$

Sehingga

$$\begin{aligned} Y(z)(1 - 0.8187z^{-1})(10.7408z^{-1}) &= X(z)(1 - 0.8966z^{-1}) \\ Y(z)(1 - 1.5595z^{-1} + 0.6064z^{-2}) &= X(z) - 0.8966z^{-1}X(z) \\ Y(z) - 1.5595z^{-1}Y(z) + 0.6064z^{-2} & \\ &= X(z) - 0.8966z^{-1}X(z) \end{aligned}$$

Persamaan bedanya:

$$\begin{aligned} y(n) &= x(n) - 0.8966 x(n - 1) + 1.5595 y(n - 1) \\ &\quad - 0.6064 y(n - 2) \end{aligned}$$